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EXCITATION AND DETECTION OF ACOUSTIC WAVES IN PLASMAS

By

William A. Saxton

February 1965

Technical Report No. 462

Cruft Laboratory
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ABSTRACT

This report contains initial results of an experimental study of acoustic disturbances in weakly ionized gaseous plasmas which exist in the positive column of dc discharges. One phase of the research concerns itself with the effects of externally perturbing such a medium with acoustic waves emanating from transducers that operate in the audio and ultrasonic frequency range. The balanced of the effort is devoted to the observation of sound waves generated by natural and uncontrolled low-frequency oscillations which commonly occur in laboratory discharges.

Compact transducers incorporated into cylindrical discharge tubes provide the source of acoustic signals and serve as microphones in detecting sound waves. Modulation of electromagnetic waves by the intentionally disturbed plasmas was measured in a novel rectangular cavity whose output was detected and fed to a wave analyzer. Resultant wave-analyzer responses indicate that the electron collision frequency was modulated in addition to the electron plasma frequency, and that the variations in both were proportional to the magnitude of loudspeaker diaphragm deflection, as predicted by simple acoustic theory. Measurements indicate that $\Delta N_e/N_e = \Delta N/N$ (where N_e and N are electron and neutral molecule densities, respectively) for slightly ionized gases which were subjected to low-frequency pressure variations in the order of 10^{-5} Torr at ambient equilibrium pressures in the 100 - 300μ range, and that such variations produced plasma-frequency perturbations of 0 - 140 kc/s at plasma frequencies up to 1000 Mc/s.

Early observations of intentional acoustic perturbations were made in stable and quiescent discharges over a pressure range for which large spontaneous oscillations in the positive column were absent. However, frequent and unpredictable appearance and disappearances of moving striations made it extremely difficult to maintain a quiet plasma and, as measurements progressed, there were indications that the plasma was often in a quasi-stable condition at which time the sound waves transmitted into the medium by the transducer actually excited a low-frequency oscillation and interacted with it. Ultimately, a separate investigation was conducted to determine the nature of the interaction between the spontaneous plasma oscillations and the sound waves generated by the transducer. In addition to an attenuation of the sound waves propagating into the plasma, a clear cut frequency mixing was observed between the low-frequency oscillations and the forced sound wave.

Finally, the transducer was used as a microphone without the RF circuitry to "listen" to the sound waves created by the moving striations in the neutral gas. Although it was impossible to correlate the magnitude of the low-frequency oscillations with the intensity of the sound waves produced by them, a one-to-one frequency correspondence was in evidence at all times. Thus, a link was established between the electrical oscillations in the plasma and acoustic waves in the neutral gas; and, a technique was demonstrated for quantitatively investigating the relationship between the two.

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1. INTRODUCTION

For almost a century there has been a great deal of research on noise and oscillations in both laboratory and natural plasmas, with the major emphasis on those perturbations which are self-excited by the medium itself. Although experimenters began reporting observations of spontaneous fluctuations in supposedly steady dc discharges as far back as 1874, the main activity in the investigation of unsteady behavior in plasmas began during the 1920's and led to the classic work by Tonks and Langmuir [1] on the theory of electron and ion plasma oscillations in ionized gases. Since that time, numerous papers have reported measurements on various types of oscillations and possible explanations for their causes [2].

In the past, several diagnostic techniques have been used to observe oscillations in the plasmas generated by laboratory discharges in the hope of relating the complex phenomena that take place to effects which occur in natural plasmas such as the ionosphere. Furthermore, experimentalists have devoted much effort to methods for perturbing plasmas, to studies of resultant electron- or ion-density variations, and to the influence of perturbations on the propagation of electromagnetic and electroacoustic waves. Recently, however, several investigators turned their attention to methods for intentionally exciting acoustic waves in ionized gases, and observations of their effect on the medium in which they propagate. Goldstein, Roux, and Dayton [3], for example, used high-power RF pulses in low-pressure gases to produce acoustic waves, while Alexeff and Jones [4] have reported the direct measurement of ionic sound-wave velocity in an acoustically disturbed plasma.

Menzel [5], in reviewing the various interactions that take place in the ionosphere because of nonlinear processes, pointed out that acoustic disturbances from rockets, missiles, explosions, and collisions of solar ion clouds with the upper atmosphere are all potential sources of acoustic waves which can impress modulation on electromagnetic waves, and asked whether or not the various kinds of interaction which result could be studied experimentally. Related to this is the establishment of a link between electromagnetic-acoustic interactions and the results of workers such as Kino and Allen [6] who have shown theoretically and experimentally that there is strong amplitude and phase modulation of signals traveling through plasmas which have been electrically disturbed by low-frequency variations.

Further interest in the acoustic properties of plasmas arose when Drummond [7] extended an ion-wave theory to include nonlinearities and showed it possible to pass electromagnetic waves through acoustically disturbed plasmas at RF frequencies below the plasma frequency where a negative dielectric constant ordinarily forbids the propagation of such waves.

If an acoustic transducer were used as the source of such a disturbance in an isotropic plasma, one could experimentally investigate the range of validity in Drummond's theory and the possible application of such a technique to the problem of communicating with space vehicles as they re-enter the earth's atmosphere during the time when the plasma sheath which surrounds them often causes a communications black-out.

By observing the effects of intentional acoustic perturbation in ionized gases, it may be possible to learn more about a plasma's complex oscillation and propagation mechanisms. With this as a long-range goal, an investigation has begun in which the perturbing devices are acoustic transducers operating in the audio and ultrasonic frequency ranges. Since such transducers also can operate passively as microphones to detect sound waves generated by the plasma itself, the initial objective was two-fold: to demonstrate the feasibility of acoustically modulating a plasma with a transducer, and to determine the usefulness of a transducer as a probe for studying natural low-frequency oscillations in a plasma.

Electrically neutral, slightly ionized isotropic gases, consisting of electrons, an equal number of ions per unit volume, and neutral molecules far in excess of electrons and ions, can be conveniently characterized by an electron plasma frequency

$$f_p = \frac{e}{2\pi}\sqrt{\frac{N_e}{m\epsilon_o}}\,, \qquad \left(\begin{array}{l}\text{e and m are the electronic charge and mass,}\\ \text{respectively, } \epsilon_o \text{ is the permittivity of}\\ \text{free space, } N_e \text{ is the electron density.}\end{array}\right) \tag{1-1}$$

and an electron collision frequency ν_c – in collisions per second – which describes the directed momentum transfer as a result of collisions between electrons and the other particles in the medium. In a simple model of such a gas, the properties of an electromagnetic wave passing through it depend upon f_p and ν_c through the complex dielectric constant

$$\underline{\xi} = \epsilon - j\frac{\sigma}{\omega} = \epsilon_o \left(1 - \frac{\omega_p^2}{\nu_c^2 + \omega^2} - j\,\frac{\omega_p^2 \frac{\nu_c}{\omega}}{\nu_c^2 + \omega^2} \right) , \qquad (1\text{-}2)^*$$

where ω is the electromagnetic angular frequency, ϵ is the dielectric constant, σ is the conductivity, and $\omega_p = 2\pi f_p$ [11]. Therefore, a perturbation in f_p and ν_c can, in principle, be detected by considering the plasma as a medium with varying dielectric constant and conductivity, and observing the modulation of electromagnetic signals propagating in it.

Although Sessler and Schroeder [8] experimentally observed the interaction between noise generated by a dc discharge and externally applied sound waves, their interest was primarily in the acoustics of the problem and no electromagnetic waves were involved. In any event, they did show that discharge noise in some frequency bands was either reduced or increased by sound waves penetrating the discharge at frequencies in the audio range. Sodha and Palumbo [9] analyzed the modulation of transmitted and reflected waves at an interface between a plasma and free space theoretically, and showed that, if any unmodulated electromagnetic wave is incident upon such an interface in a direction opposite to the incidence of an acoustic wave, the reflected electromagnetic wave will be modulated at the acoustic

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*Assuming $e^{j\omega t}$ time dependence frequency. The geometry of this analysis was entirely different from that which exists in a closed, bounded discharge tube, but measurements in the latter situation could be useful in confirming their assumptions about the composition of the gas during an acoustic cycle.

Collision frequency depends, among other factors, upon gas pressure. Similarly, the electron density in a discharge-tube plasma is a function of the density of neutral molecules so that, in this case, the plasma frequency is also pressure dependent. Thus, it seemed reasonable to expect that the pressure variations produced by sound waves impinging upon a plasma would cause time-varying alterations in the complex dielectric constant of the medium. In the research reported here, a slightly ionized plasma has been created in the laboratory by a continuous dc discharge and disturbed by the same type of acoustic standing waves that exist in a closed cylindrical pipe driven at one end by a piston. The plasma is treated as a time-varying lossy dielectric post in a specially designed microwave cavity which surrounds it, and any perturbations in f_p and ν_c are seen as a modulation of the cavity output. Thus far, argon, argon-neon mixtures, and neon have been used as media in the tubes at pressures ranging from 0.1 to 30 Torr. However, most of the measurements have been in neon at pressures of 100 to 300 microns.

Attempts to observe time variations in the dielectric constant of the acoustically disturbed plasma have been greatly hindered by spontaneous low-frequency oscillations which commonly appear in dc discharge tubes over wide and well-defined ranges of currents and pressures [10]. Because such oscillations interact with the sound waves transmitted into the plasma, represent one of the most frequently observed of the natural oscillations occurring in a gas-discharge plasma, and are not fully understood, they were of more than secondary interest. Often referred to as "moving striations," they manifest themselves as periodic variations in the terminal voltage and current of a discharge. Their fundamental frequencies range from 200 or 300 c/s to tens, or even hundreds, of kc/s depending on the parameters of the discharge and the geometry of the tube. Harmonics of such fluctuations typically have rms magnitudes up to several percent of the average discharge current, and the oscillations can be easily detected optically using phototubes, or electrically with Langmuir probes, external capacitive coupling, and, in the simplest scheme, an oscilloscope across a series resistor in the external circuit of a discharge tube. However, it seemed likely that the low-frequency oscillations would also be observable acoustically with a sensitive microphone. Therefore, a transducer was developed in which, for maximum sensitivity, the diaphragm was wholly inside the discharge tube.

Initial efforts were directed toward the design and construction of compact transducers for installation into the cylindrical discharge tubes that were used for the production of practical laboratory plasmas. Two units were built, one a variation of a solid-dielectric electrostatic type, the other a bipolar moving-armature transducer for use in the low audio range. Early experiments showed that both produced observable perturbations in varying degrees when operated as loudspeakers, although the sound intensities generated by the former were insufficient for reliable measurements. In addition to providing sizable and easily measured acoustic variations in the electron density and collision frequency of a plasma, the moving-armature transducer was used as a loudspeaker at audio frequencies to indicate a significant interaction between acoustic waves launched into unsteady discharges and natural plasma oscillations. Furthermore, the transducer has been successfully employed as a microphone to detect the spontaneous oscillations (in the absence of a loudspeaker signal) and thus establishes a potentially valuable diagnostic tool which may shed new light on the nature of "moving striations."

Of the several transducer mechanisms which presently exist, most of them are inappropriate for the excitation and detection of acoustic waves in plasmas. Section 2 considers the requirements for this application and contains the important acoustic and constructional specifications. After pointing out the limitations of conventional designs, details of a satisfactory moving-armature transducer are presented along with a brief description of its measured performance.

Section 3 presents the basic experimental configuration. Full details of the cavity, discharge tube, and circuitry are given along with an outline of a versatile vacuum system used in tube cleaning and processing. Also included is the theory leading to a method for conveniently measuring plasma frequency and collision frequency in an ionized medium.

Details of the measurements, significant data, and results appear in Section 4. In addition, possible explanations for the observations are advanced that tend to support theoretical estimates of the plasma perturbation effect which are derived by means of simple acoustic theory and an elementary model for the plasma. Because the low-frequency plasma oscillations, or moving striations, represent such an important influence in the work, a brief summary of their characteristics is also included and used to explain deviations of certain data from what would be expected in a well-behaved plasma.

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2. TRANSDUCER DESIGN AND DEVELOPMENT

In addition to possessing acceptable acoustic characteristics, transducers used for the excitation and/or detection of acoustic waves in laboratory plasmas must satisfy two basic requirements. First, they must be compact and dimensionally compatible with the structure that contains the plasma. Second, they must be made of materials which do not out-gas in high-vacuum environments, and are capable of withstanding temperatures of 500°C. These conditions are necessary to create a reproducible plasma which is relatively free of contaminants.

Of the several transducer mechanisms which presently exist, most of them are inappropriate for use in plasmas because they fail to meet one, or both, of the basic requirements. This section considers such requirements and establishes the important acoustic and constructional specifications for transducers which are housed in long, cylindrical glass discharge tubes approximately 2 - 1/2" in diameter. After pointing out the limitation of conventional designs, details of a satisfactory moving-armature transducer are presented along with a brief description of its measured performance. Although the moving-armature unit was used exclusively in the measurements reported here, a modified solid-dielectric type transducer has also been built and promises to be quite useful as a microphone in detecting plasma sound waves[1].

2.1 Acoustic Requirements

Continuous dc discharge plasmas used in the experiments described in this research operate mainly at pressures in the 0.1-5 Torr range. For such reduced pressures there is a sizable mismatch between the mechanical impedance of any piston-like loudspeaker or microphone and the characteristic acoustic impedance of the medium in which the unit operates. As a result, it is quite difficult either to detect or transmit acoustic power in the neutral gas under these conditions. Transducers normally operate at atmospheric pressure and, although their characteristics are well known in this case, to predict accurately their behavior at reduced pressures is not an easy matter. The literature is not very helpful in estimating such performance, since most of the work at low pressures has been directed toward determining attenuation and phase constants for acoustic waves in various gases and not on power transfer or sound intensity in the media.

Using elementary acoustic theory for sound propagation in a closed-end cylindrical pipe with a rigid reflector at one end, and driven at the other end by a piston, one can estimate relative pressure variations in the resultant standing waves; such variations are directly proportional to piston deflection at a given point for a fixed driving frequency. Calculations indicate that relative pressure perturbations of 10^{-4} are possible at standing-wave maxima in the gases that are normally used in discharge tubes provided that peak-to-peak piston deflections of 0.005"-0.010" are realized. If this is the case, a simple model for a plasma predicts that observable and measurable plasma effects will accompany the excitations of a neutral gas.

At attempt to estimate the sensitivity required by a transducer to detect acoustic waves generated by a plasma would be extremely difficult because the very nature and intensity of such waves in the ionized gas are still unresolved. Of all the possible plasma oscillations that could create sound waves in the medium, the most likely to be detected are those associated with spontaneous low-frequency oscillations, or "moving striations," as they are sometimes called. These large-signal perturbations in the average current of dc discharge tubes can have fundamental frequencies ranging from 200 - 2000 c/s for the plasma geometries in these experiments.

2.2 Vacuum Requirements

De-gassing, or out-gassing as it is frequently called, refers to the evolution of gases from materials which are either porous and tend to trap gas, or which have vapor pressures above their intended ambient pressures that create an "evaporation." Either situation usually makes it difficult to pump or exhaust a system containing such materials below pressures required for discharge-tube processing and operation. Even if a pumping system had the capacity to overcome the constant out-gassing of these elements, there would be a continual rise of ambient pressure once any volume containing them was valved off from the rest of the system, resulting in an undesirable time-varying environment. Furthermore, the composition of the evolved gas, be it air, water vapor, carbon dioxide, or any one of a number of other gases, would most likely be different from the gas used in any experiments so that a contamination would result. It is well known that even very small percentages of impurities in a given gas can drastically alter its properties, especially in the conditions that accompany the formation of a laboratory gas-discharge plasma. One sees this in the intentional addition of 0.2% argon to 99.8% neon in commerically available mixtures to affect an easier ignition in neon discharges. In this case, other properties of the neon gas are not seriously affected; but in general, impurities are undesirable.

To minimize de-gassing and impurities, the usual procedure in discharge-tube processing is to raise all parts exposed to vacua to temperatures between 400°C and 500°C for time periods in excess of several hours. Here, the intent is to "drive out" as much abscrbed or adsorbed gas as possible while simultaneously pumping it out of the system. Water vapor held by glass structures is one of the main targets of this procedure which minimizes undesirable gases and assures negligible contamination when the system under test cools to operating temperatures, even if there should be minor temperature rises on the glass during subsequent experimental procedures.

Such vacuum system limitations make it clear that any transducer elements located within the plasma discharge tube must be constructed of materials such as selected glass, metals, and ceramics. Unfortunately, conventional designs usually have one or more component parts made with plastics such as Mylar or Bakelite (whose melting points are well below 500°C), rubber, enamel, or other undesirable materials.

2.3 Moving-Armature Transducers

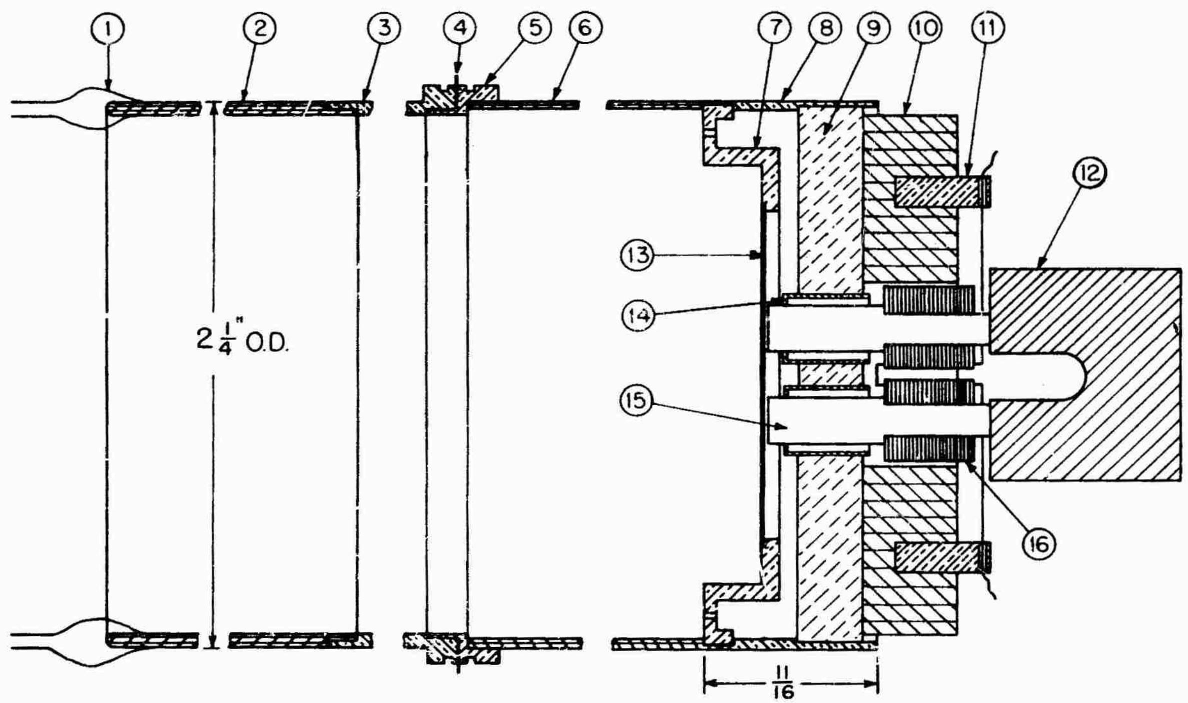
Since initial experiments in the plasma at acoustic frequencies in the audio range were planned, the bipolar moving-armature transducer (similar to that used as a receiver in common telephone handsets [2,]) merited primary consideration. Moving-armature transducers have a thin flexible diaphragm which serves as an integral part of a variable-reluctance magnetic circuit. The diaphragm acts as a return path for magnetic flux transferred across an air gap by suitably designed pole pieces which couple electromagnetically to driving coils (or pickup coils depending on whether the unit is operated as a loudspeaker or microphone). A dc biasing flux created by a permanent magnet is present in the magnetic circuit to assure that the fundamental of any driving or received signal predominates over higher harmonics when the diaphragm flexes due to the varying magnetic forces created in the air gap.

Conventional transducers with a flexible magnetic membrane have maximum responses and sensitivities at low acoustic frequencies in the audio range. Their frequency responses, either as microphones or loudspeakers, as far from "flat" and usually show marked resonances in the 800 -1500 c/s range. However, they exhibit much larger diaphragm deflection than other conventional transducers, especially at or near their resonances, making them especially suitable for reduced-pressure applications. Although with careful mechanical and electrical design it is possible to reduce, or even eliminate, the resonances – and, in fact, to make these units relatively flat as in telephone applications – the usual resonances are somewhat desirable in applications where gross effects are to be studied and calibration is not necessary.

As with other conventional transducers, the main drawback of the moving-armature construction is the normal use of plastics, enamel-coated wire insulators, and support structures which violate the vacuum requirements. Because of this, commercially-made units such as Western Electric's HA-1 should not be placed inside the discharge tube. However, the desirable acoustic properties of such a design motivated the development of a transducer which is an integral part of the discharge tube and is compatible with high-vacuum requirements.

2.3.1 Construction Details

Moving-armature transducers consist of four basic elements: a thin membrane or diaphragm made of magnetic material, pole pieces used to conduct magnetic flux, coils of wire which surround the pole pieces, and a permanent magnet to provide a biasing flux. Various designs using the moving-armature principle differ in the size and material of the diaphragm, the number, position and geometry of the pole pieces, the position and electrical impedance of the coils, the strength of the dc biasing field, the spacing between the faces of the pole pieces and the diaphragm, and, of course, the physical layout of the interrelated basic elements. In addition, there are mechanical support structures which, as a rule, are not critical with regard to acoustical performance. Among all these considerations, however, the main requirement for the plasma experiments is that the diaphragm be located within the discharge tube. This made it possible to design a unit whose housing was an extension of the discharge tube itself, with the diaphragm excited by pole pieces which were vacuum-sealed into the end of the tube, and excited outside of it by the accessible and easily removable voice coils and permanent magnet (Figs. 2-1 and 2-2).



Bill of Material		
No.	Part	Amt.
1	KOVAR-GLASS SEAL	1
2	KOVAR SLEEVE	1
3	COPPER EXTENSION	1
4	CUPRONICKEL WELD-RING	1
5	COPPER ADAPTER	1
6	KOVAR SLEEVE	1
7	CUPRONICKEL SUPPORT	1
8	CUPRONICKEL SLEEVE	1
9	CERAMIC BACKPLATE	1
10	PHENOLIC TERMINAL BOARD AND COIL SUPPORT	1
11	BRASS TERMINAL	2
12	PERMANENT MAGNET	1
13	VANADIUM PERMENDUR DIAPHRAGM	1
14	CUPRONICKEL ADAPTER	2
15	INGOT-IRON POLE	2
16	DRIVING COIL	2

Fig. 2-1. Construction details of moving-armature transducer used in plasma excitation and detection experiments. A large Kovar-glass seal makes this transducer an integral part of the discharge tube.

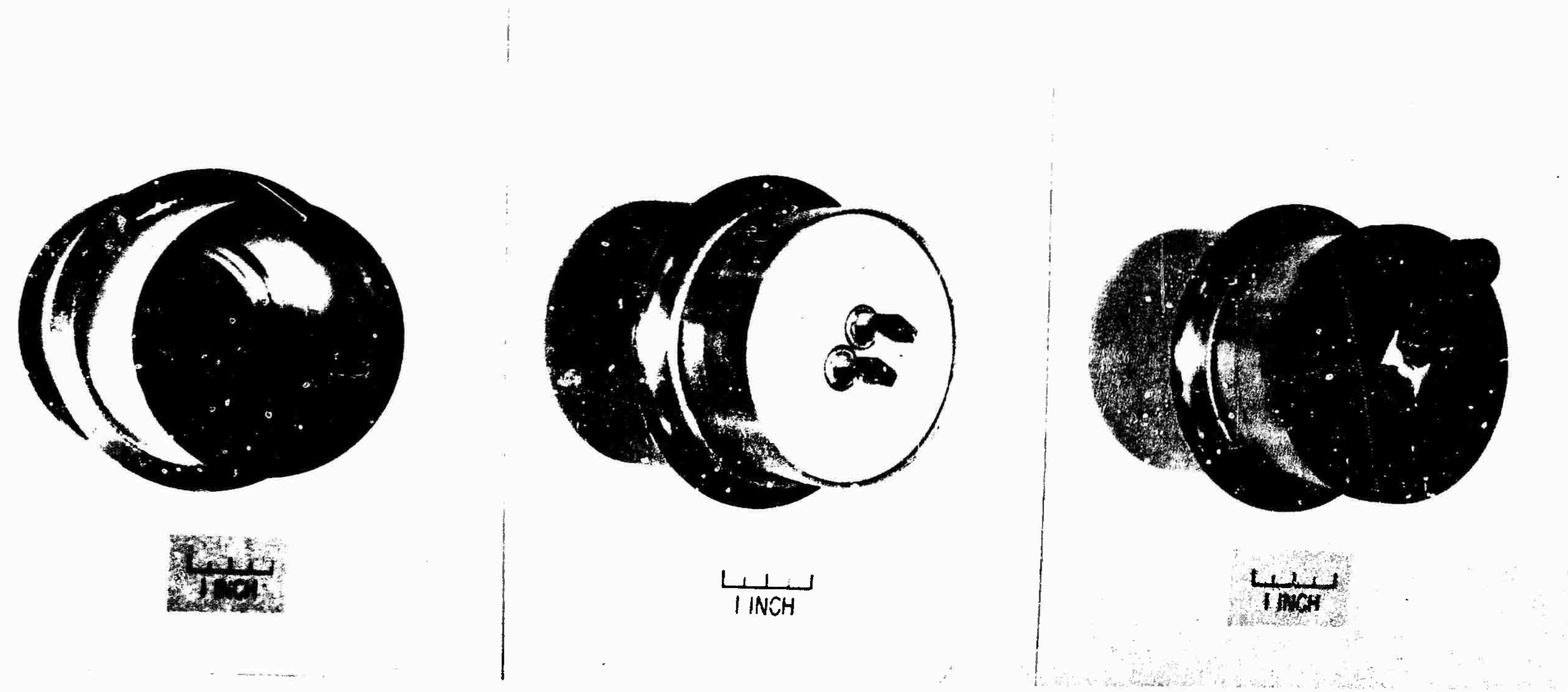


Fig. 2-2. Experimental moving-armature transducer. View at left shows pole pieces protruding through ceramic backplate into discharge tube; diaphragm (not shown in photograph) rests on innermost shoulder. In back view of unit (center), coil assembly and permanent magnet have been removed. At right, coil assembly is in position around pole pieces.

For many years the Bell Telephone Laboratories have been working to improve the acoustic characteristics of the bipolar moving-armature transducer which is used as the receiver in the ordinary household telephone. One such design, known as the HA-1,* provided the basis of the unit built for the plasma application. The HA-1 has an excellent frequency response up to approximately 3000 c/s, and a fairly linear diaphragm displacement up to about 4.5 volts input to the driving coils. However, these characteristics are based upon an environment meant to simulate the chamber and loading of the telephone itself, and it is quite difficult to extrapolate measurements under these conditions to those which would be encountered by the receiver as it propagates sound waves down a cylindrical discharge tube with a closed end. Under normal operating conditions in the telephone, the HA-1 has a diaphragm deflection of $1\text{-}2 \times 10^{-6}$ inches and develops a pressure of 3 dynes/cm^2 for diaphragm swings 4 or 5 times that amount [3]. These figures are for applied driving voltages of only 15 mv and, based upon this fact, it appears that diaphragm deflections of several thousandths of an inch would be possible with much higher driving voltages, even though there might be a great deal of harmonic distortion. In any event, simple acoustic theory indicates that the key to sizable plasma perturbations is the diaphragm deflection so that, for an initial examination of gross effects, linearity and harmonic distortion as a function of acoustic frequency are not major considerations.

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*Produced by Western Electric. Driving coils have nominal input impedance of 135 Ω (with a phase angle of 42°) at 1000 c/s. Measured impedance of coils at dc is approximately 27 Ω.

Basically, the development of a high-temperature low-pressure transducer patterned after the HA-1 involves materials technology more than anything else. Although it was known a priori that even the slightest departures from the HA-1 design would cause drastic modifications in the acoustic characteristics, the basic assumption was that the diaphragm deflection could be made to have the same order of magnitude as in the Western Electric unit. Many of the component parts from an actual HA-1 were used in the final version. The diaphragm, made of vanadium permendur (49% cobalt, 49% iron, 2% vanadium), and the coil assembly were usable; pole pieces were fabricated from Armco ingot iron in contrast to the Permalloy alloy used in the HA-1. However, the major obstacle in adapting the telephone unit for the discharge tube was in replacing the Bakelite backplate which held the pole pieces with a disc which would not only serve this purpose, but would seal the end of the discharge tube as well. Since a high-temperature material was required, an alumina ceramic disc was finally developed along with the appropriate sealing and brazing materials and processes. The design and installation of this ceramic disc was perhaps the most difficult problem in the entire transducer fabrication.

Although no quantitative testing was done on the transducer prior to putting it into the plasma, it was apparent from the start that the acoustic output was a very sensitive function of the spacing between the pole pieces and the diaphragm. The final spacing of 0.010" was determined by a cut-and-try procedure after the fabricated transducer had been sealed into one half of the glass discharge tube. With a very crude sound probe – the ear – as a gauge, this dimension seemed to be the optimum in terms of sound intensity. Even small departures from this dimension made a pronounced difference in sound power, even granting the crudeness of the "measurement."

It would be difficult to list all of the engineering difficulties encountered in developing this unit. Most of them were a result of the incompatibility of sealing, brazing, welding, and glass-blowing techniques. For example, since Kovar is required for glass sealing, one major problem was to find a way to vacuum-seal the transducer housing with a Kovar-to-glass assembly without heating the ceramic through heat conduction to a point where it would crack or rupture at its housing seal. This was finally done by Heli-arc welding of a previously, and separately prepared, Kovar-glass seal to the transducer housing with a cupronickel ring as a fusing material. This technique was developed after RF-induction brazing had cracked the Kovar-to-glass seals and the ceramic on separate occasions, even though heat conduction paths were made as long as possible. Problems such as these were frequent and required a great deal of advance thinking in the fabrication procedure. Full details of each important component part for the transducer appear in Fig. 2-3.

2.3.2 Acoustic Characteristics

To evaluate quantitatively the performance of any transducer in the discharge tube under reduced pressures, it would be necessary to know the actual pressure variations caused by it at each point along the column in the tube. Ideally, one should use a sound probe to measure such pressures at each point of interest, bearing in mind that the closed tube will produce

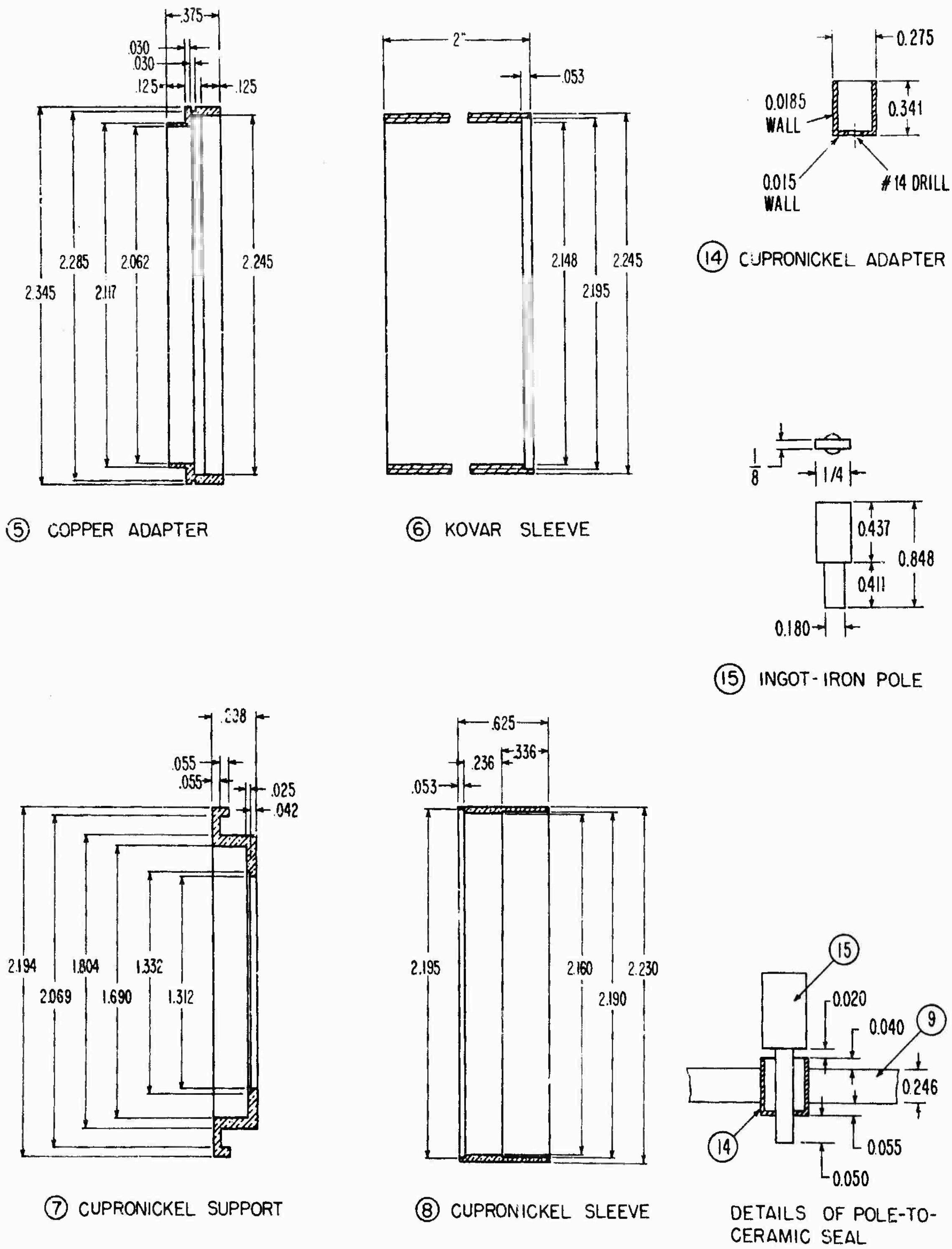
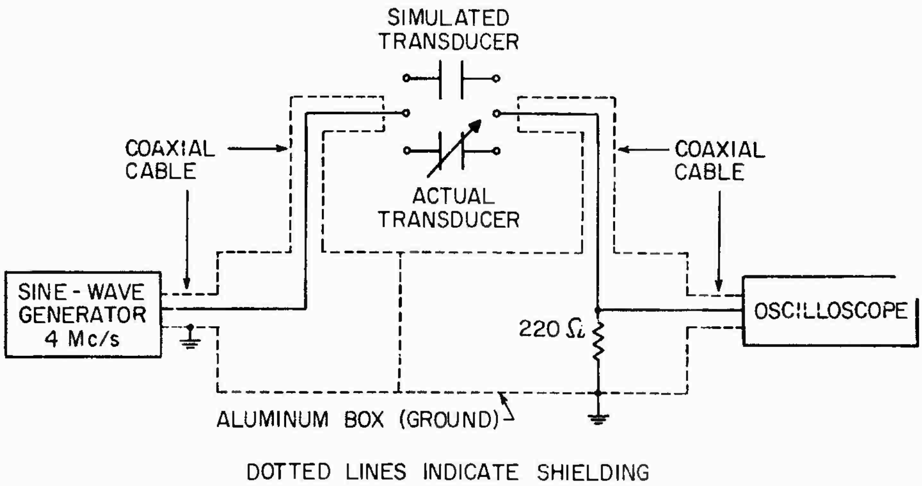


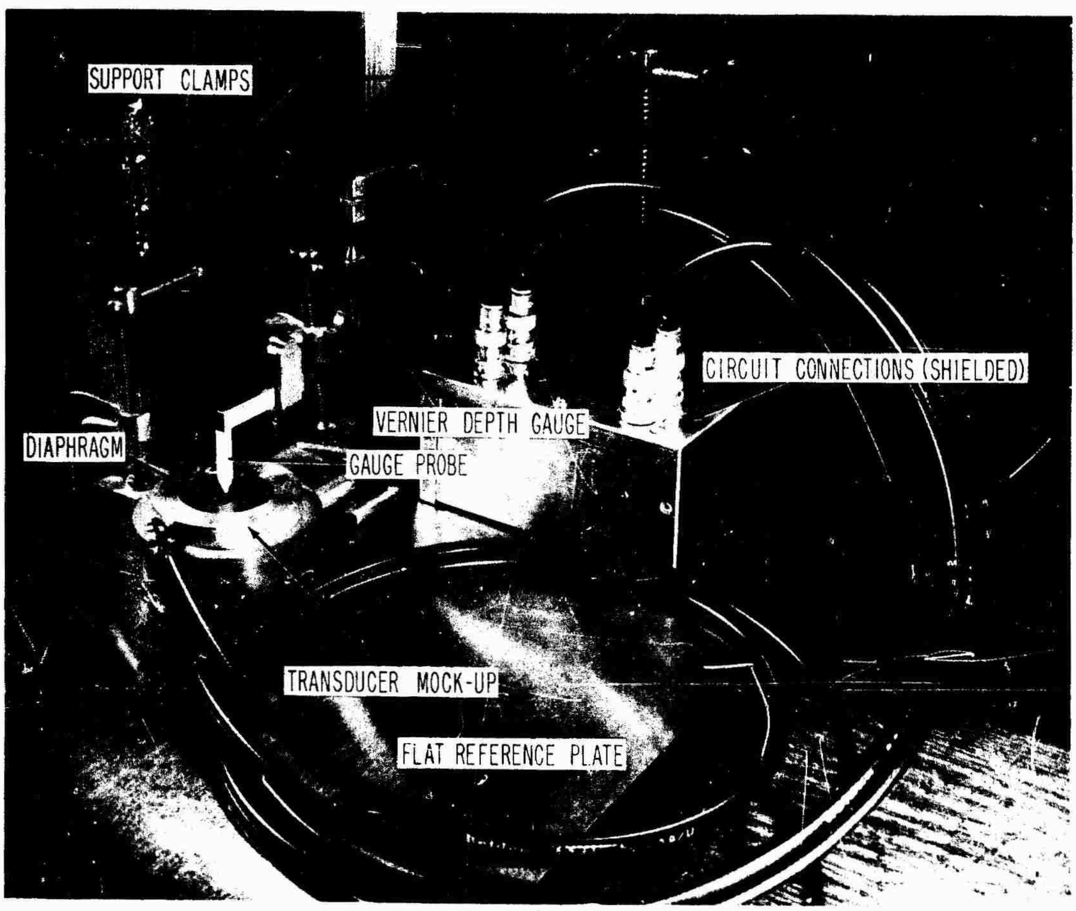
Fig. 2-3. Complete dimensions for component parts of moving-armature transducer.

standing acoustic waves. Unfortunately, this is not easily done without disturbing the discharge itself unless, of course, the microphone or sound probe is also the discharge anode, and one uses the measured pressure at the anode to infer the pressure at any point in front of it through the anticipated standing-wave pattern. Since no such acoustic probe was used in the construction of the initial discharge tubes, the only other way to estimate small changes of pressure produced by the bipolar moving-armature transducer was to measure the magnitude of its diaphragm deflections under the conditions actually encountered in the tube. Measurements of this nature under any other environmental conditions would be considered inconclusive.

Optical methods are frequently used to observe and measure minute diaphragm displacements in transducers. However, they make it necessary to illuminate some spot on the diaphragm surface with a beam of light and, since the construction of the transducer makes it impossible to see the diaphragm visually, this approach is unacceptable. As an alternative to this technique, a substitution procedure was used whereby the capacitance between the pole pieces and the diaphragm of the actual transducer was compared to that of an electrical equivalent outside the discharge tube. The simple circuit of Fig. 2-4 (a) was used to observe the modulation of a high-frequency signal created by the varying capacitance of the transducer when in actual operation. By means of an oscilloscope, the depth of the current modulation was compared to the current variation produced by a static deflection of the diaphragm in a substituted mock-up with a very sensitive vernier depth gauge resting on the center of the diaphragm as in Fig. 2-4 (b).



(a)



(b)

Fig. 2-4. (a) Circuit used for diaphragm-deflection measurement by substitution method. (b) Actual measurement setup.

A first step in the procedure was to balance the actual transducer against the prototype by adjusting the vernier gauge to the same level as that observed for the actual unit with the diaphragm in its rest position. This compensated for the static deflection created by the pull of the permanent magnet on the diaphragm in the tube. For a given frequency and applied driving voltage, the diaphragm swings back and forth to create some modulation – usually sinusoidal except under highly nonlinear conditions – whose minimum corresponds to the point where the diaphragm comes closest to the poles. Then the leads connected to the discharge-tube unit are connected to the prototype and its diaphragm is deflected to a dc level corresponding to the previously observed valley. Of course, with the setup used it is impossible to deflect the diaphragm of the mock-up to correspond to the maximum departure of the actual diaphragm from the pole pieces. However, the deflection on the transducer in the tube is assumed to be symmetrical about the rest position, an assumption borne out by the modulation symmetry for all but the highest driving voltages.

By means of this procedure, diaphragm deflection curves similar to those in Fig. 2-5 were produced. Note the reasonably good linearity in the diaphragm deflection for small driving voltages and the leveling off, or saturation, for extremely high values. This is typical of this type of transducer and was expected in view of the performance cited by Bell Labs for the HA-1. Of particular interest are the rapid increases of diaphragm deflection for certain acoustic frequencies as the driving voltage increases. This is probably due to mechanical nonlinearities in the disc itself at or near

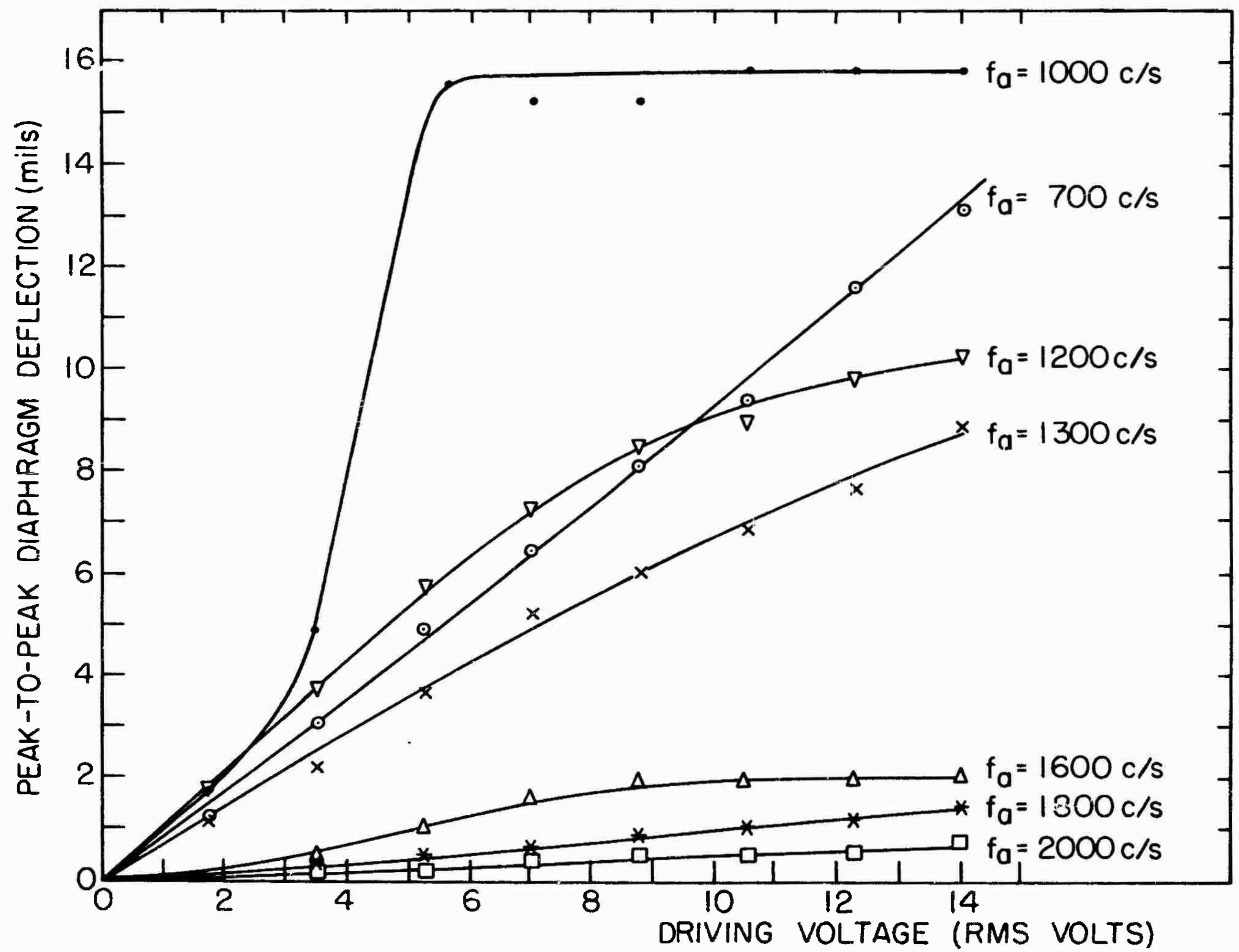


Fig. 2-5. Diaphragm-deflection characteristics for moving armature transducer when operated as loudspeaker. Abscissa is voltage applied directly to coils.

the resonant frequencies when the deflections of the diaphragm are extremely large. This, too, has been observed in other units and is related to phenomena which occur when the diaphragm's proportional limit is exceeded. Since the main resonance of the unit was qualitatively determined to be somewhere in the vicinity of 1000 c/s, it is quite likely that this is the correct explanation for the effect observed in the 1000 c/s curve of Fig. 2-5.

Meaningful measurements at lower acoustic frequencies than those in Fig. 2-5 were all but impossible due to the complex motion of the diaphragm as evidenced by the distortion of the waveform observed on the oscilloscope. At these frequencies, more than one vibration mode was probably excited in addition to the noise created by the unclamped plate banging against the transducer housing and, in some cases, against the pole pieces themselves.

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3. APPARATUS AND TECHNIQUES

3.1 Basic Experimental Configuration

Initial measurements were made in a glass T-shaped cylindrical discharge tube which was placed with its axis parallel to an RF electric field and perpendicular to the direction of wave propagation. The field was maintained in a rectangular cavity 6 feet long with a cross section of 1" x 12" (Fig. 3-1). The discharge tube pierced the center of the cavity through a 2 - 5/8" hole placed so that the narrow dimension was in the positive column of the discharge. Propagation of acoustic waves took place along the axis of the tube at frequencies whose wavelengths in the plasma were greater than one inch so that the cavity probed a more-or-less uniform electron density distribution in the direction of the electric field when the plasma was perturbed.

Plasmas were created in the tube by a continuous dc discharge which occurred between a hot oxide-coated cathode in the T-extension and an anode whose position was adjustable by moving a soft iron slug with an external magnet. This movable anode provided additional flexibility in varying the characteristics of the discharge and, in addition, served as a variable reflector for sound waves propagated along the axis of the tube from a transducer located at the opposite end. In the early experiments, a solid-dielectric type transducer was mounted completely within the discharge tube with only its two leads protruding from the end through a common tube-press. However, this transducer was ultimately replaced by the moving-armature unit which was attached to the far end of the discharge tube with a large Kovar-glass seal.

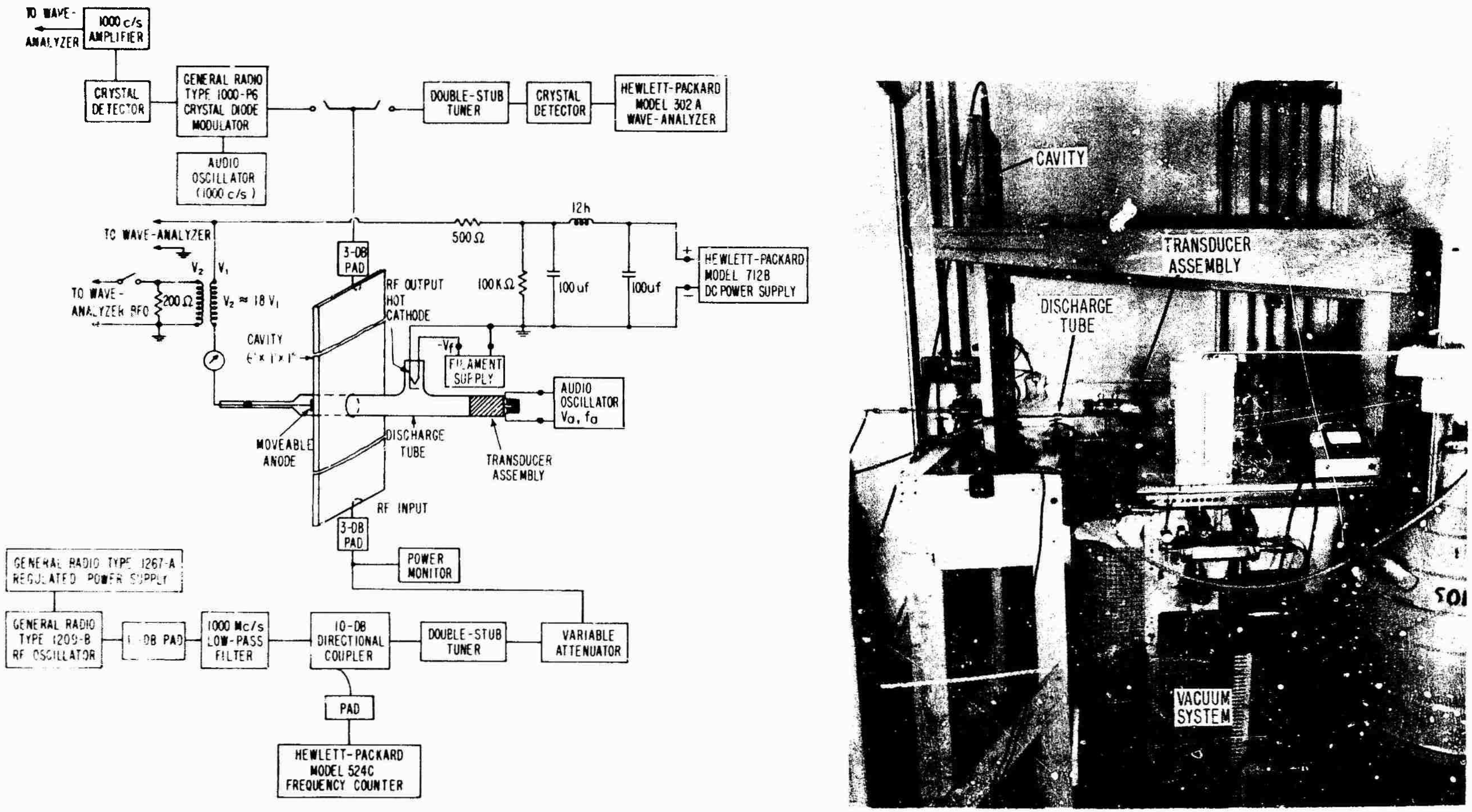


Fig. 3-1. Physical layout and circuitry in the system for exciting and detecting acoustic waves in plasmas.

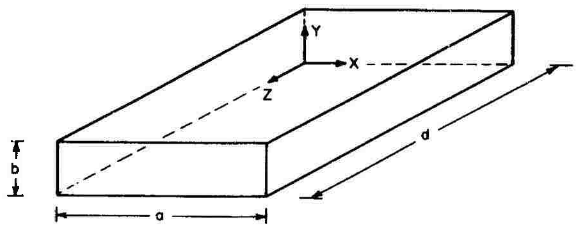
3.2 Construction of the Cavity

Copper sheet, 1/8" thick, was used in the construction of the rectangular cavity. Four aluminum right-angle strips mounted along each of the long edges held the cavity walls together. Because of the natural tendency of the walls to sag under the weight of the copper itself, it was necessary to use four support braces on each of the two large cavity walls to maintain a 1" separation between the opposite walls throughout the cavity. After one end plate had been screwed into position, all joints in the cavity were filled with a highly conducting silver paint. Then, the second end plate was screwed into position following the application of silver paint to its joints, thus completely sealing and closing the cavity. An RF coupling loop 3/4" in diameter was connected to the center post of an N-type connector and soldered to each end plate. It was oriented in such a way as to excite or detect TE modes. The ends of the cavity were identical and interchangeable so that they could be used for driving and receiving.

The dimensions of the cavity were determined from the anticipated plasma frequencies and the desire to use resonant modes. These would make it possible to apply simple perturbation techniques in determining the parameters of the undisturbed plasma by measuring changes in the cavity resonance and Q. The 1" dimension was based on the expected sound wavelengths in the discharge. For fairly uniform RF fields at the center of the cavity, its width and length had to be much larger than the dimensions of the discharge tube. On the other hand, for the expected 200 to 1000 Mc/s plasma frequencies, TE modes ranging from the TE_{101} mode, resonant at approximately 500 Mc/s, to the TE_{109} mode in the vicinity of 900 Mc/s have been used to insure single-mode propagation and minimal signal attenuation due to the plasma.

3.2.1 Cavity Resonances

Figure 3-2 summarizes the fields in a rectangular cavity for both transverse electric and transverse magnetic modes. To determine the possible modes for an exciting loop placed at x = a/2 in the a x b cross section, consider the geometry of 3-2(b). The loop excites H_x at its maximum so that TM_{m0p}, TE_{0np}, TM_{0np}, and TE_{mn0} modes will not be excited. Furthermore, since $H_x = 0$ at $x = a/2$ when m is even, the $TE_{m_e np}$ and $TM_{m_e np}$ modes are not excited, where m_e represents an even integer. Of the remaining possible modes, $TE_{m\alpha p}$ and $TM_{m\alpha p}$ for $\alpha \geq 1$ can only propagate at frequencies greater than about 5000 Mc/s in a cavity with the given dimensions. Because TM_{m0p} modes are not excited with the coupling loop used, only TE_{m0p} modes (with m odd) are of interest. Table 3-1 summarizes calculated resonant frequencies of several of the usable modes in the cavity and compares them to the resonances actually measured. Also included in Table 3-1 is a summary of the measured resonances when the holes through which the discharge tube protrudes are completely open. For measurements of the cavity characteristics without the discharge tube, brass plugs 1/8" thick fit tightly into the holes to provide a completely closed cavity. Discrepancies between the calculated and measured resonant frequencies of the closed cavity were probably due to the sag in the copper sheet used to form the cavity walls. Although support braces were used to minimize the sag, there was still a slight non-uniform departure from the desired 1" dimension.



TE_{mnp} mode:

$$E_x = \frac{j\omega\mu C}{k_c^2}\left(\frac{n\pi}{b}\right)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{a}\sin\frac{p\pi z}{d}$$

$$E_y = -\frac{j\omega\mu C}{k_c^2}\left(\frac{m\pi}{a}\right)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\sin\frac{p\pi z}{d}$$

$$H_x = -\frac{C}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{m\pi}{a}\right)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\cos\frac{p\pi z}{d}$$

$$H_y = -\frac{C}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{n\pi}{b}\right)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\cos\frac{p\pi z}{d}$$

$$H_z = C\cos\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\sin\frac{p\pi z}{d}$$

TM_{mnp} mode:

$$E_x = -\frac{D}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{m\pi}{a}\right)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\sin\frac{p\pi z}{d}$$

$$E_y = -\frac{D}{k_c^2}\left(\frac{p\pi}{d}\right)\left(\frac{n\pi}{b}\right)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\sin\frac{p\pi z}{d}$$

$$E_z = D\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\cos\frac{p\pi z}{d}$$

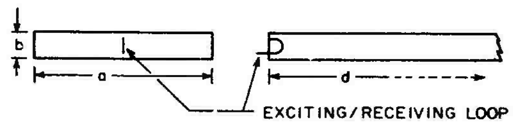
$$H_x = \frac{j\omega\epsilon D}{k_c^2}\left(\frac{n\pi}{b}\right)\sin\frac{m\pi x}{a}\cos\frac{n\pi y}{b}\cos\frac{p\pi z}{d}$$

$$H_y = -\frac{j\omega\epsilon D}{k_c^2}\left(\frac{m\pi}{a}\right)\cos\frac{m\pi x}{a}\sin\frac{n\pi y}{b}\cos\frac{p\pi z}{d}$$

$$k_c^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$k = \frac{2\pi}{\lambda} = \left[\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 + \left(\frac{p\pi}{d}\right)^2\right]^{1/2}$$

(a)



(b)

Fig. 3-2. (a) Field relations for TE and TM modes in a rectangular cavity. C and D are constants, μ is the magnetic permeability of the medium within the cavity, ϵ is the dielectric constant; $\omega = 2\pi f$. For the cavity actually used in the measurements, a = 12" = 0.3048 m, b = 1" = 0.0254 m, d = 72.125" = 1.832 m. (b) Orientation of exciting/receiving loops.

Mode	Calculated Resonant Frequency (Mc/s)	Measured Resonant Frequency With Plugs (Hole Closed) (Mc/s)	Measured Resonant Frequency Without Plugs (Hole Open) (Mc/s)
TE_{101}	498.9	490	491
TE_{102}	518.7	509	509
TE_{103}	550.0	540	543
TE_{104}	591.2	581	580
TE_{105}	640.2	633	638
TE_{106}	695.4	688	685
TE_{107}	755.5	735	745
TE_{108}	819.3	815	808
TE_{109}	886.2	—	880
TE_{1010}	955.3	—	—
TE_{1011}	1026.4	—	—

Table 3-1. Calculated and measured resonances in rectangular cavity.

However, in the final experimental configuration, the cavity is not completely closed, the holes are open, and a complex pattern of fringing fields occurs at the holes.

3.2.2 Measurement of Plasma Frequency and Collision Frequency

Plasma and collision frequencies in the positive column of the discharge are determined by the cavity perturbation technique in which a modulated CW cavity output is used to measure the shift in the cavity resonance and change in Q due to the presence of the plasma. If the plasma is treated as a dielectric post creating small perturbations in the RF field, and a Bessel function radial distribution of electron density is assumed in the cylindrical discharge tube, it is possible to relate such measurements to f_{pc} — the plasma frequency at the center of the tube — and ν_c. Detailed derivations of the shift in resonance frequency as a function of the small changes in the medium filling a cavity can be found in the literature [1]. They are summarized here for a cylindrical plasma post in a rectangular cavity.

Figure 3-3 (a) shows a completely-closed perfectly-conducting cavity filled with a medium characterized by a complex dielectric constant $\underline{\xi}$ and a magnetic permeability μ_0 equal to that of free space, and which supports fields $\vec{E}_0$ and $\vec{H}_0$ (arrows indicate vector quantities). Maxwell's equations relating $\vec{E}_0$ to $\vec{H}_0$ are

$$-\nabla \times \vec{E}_0 = j\omega_0 \mu_0 \vec{H}_0 , \tag{3-1}$$

$$\nabla \times \vec{H}_0 = j\omega_0 \underline{\xi} \vec{E}_0 , \tag{3-2}$$

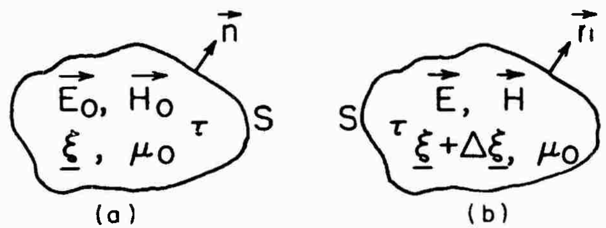


Fig. 3-3. General cavity.

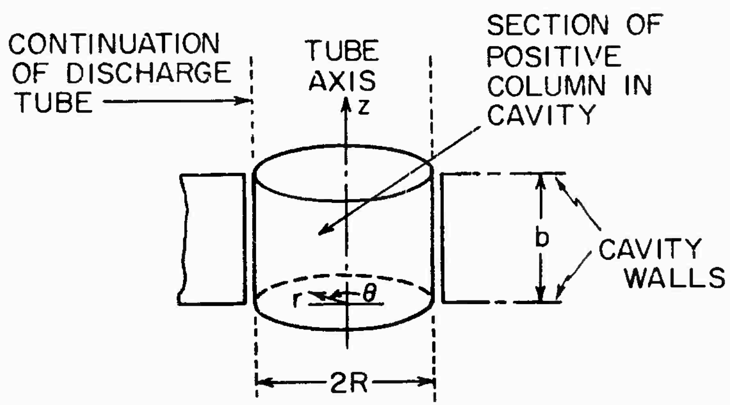


Fig. 3-5. Discharge tube in flat rectangular cavity.

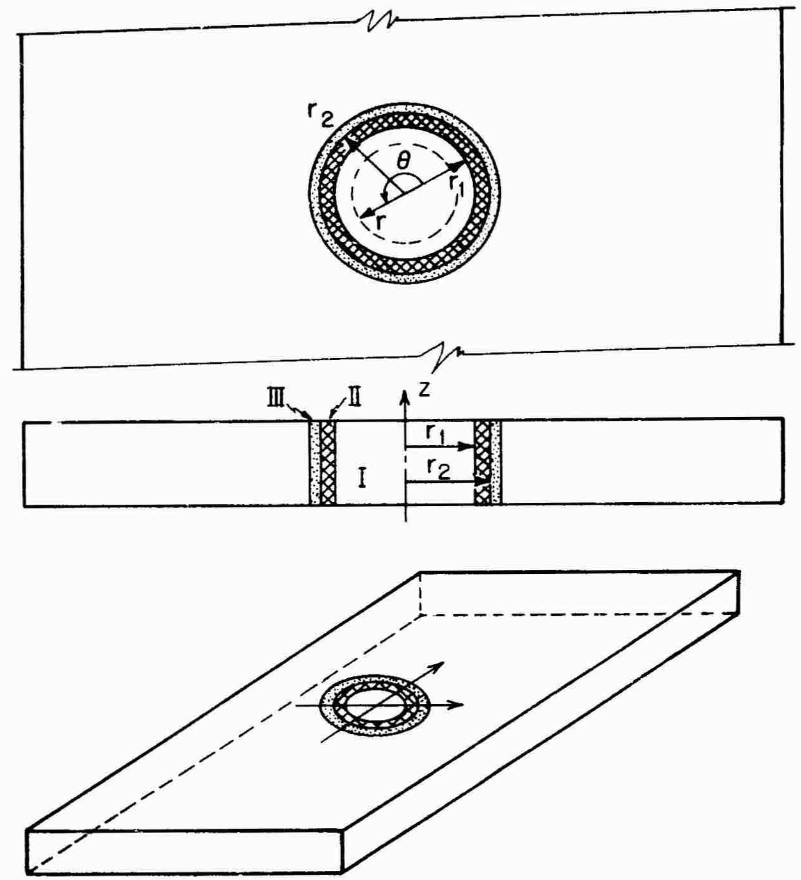


Fig. 3-4. Glass-enclosed plasma post surrounded by hole in walls of rectangular cavity. This geometry approximates the experimental setup in which a cylindrical glass discharge tube protrudes through the cavity; the hole is 2-5/8", slightly larger than the discharge tube. Region I: plasma. Region II: glass. Region III: air.

where the fields are assumed to have an $e^{j\omega t}$ time dependence. Figure 3-3(b) shows the same cavity in which the complex dielectric constant of the medium has been perturbed by an amount $\Delta\underline{\xi}$, resulting in fields $\vec{E}$ and $\vec{H}$ related through Maxwell's equations as follows:

$$-\nabla \times \vec{E} = j\omega\mu_0 \vec{H}, \tag{3-3}$$

$$\nabla \times \vec{H} = j\omega (\underline{\xi} + \Delta\underline{\xi}) \vec{E}. \tag{3-4}$$

Dotting the conjugate of $\vec{E}_0$ [denoted by $\vec{E}_0^*$] into (3-4), and dotting $\vec{H}$ into the conjugate of (3-1) gives

$$E_0^* \cdot \nabla \times \vec{H} = j\omega(\underline{\xi} + \Delta\underline{\xi}) \vec{E}_0^* \cdot \vec{E} \tag{3-5}$$

and

$$-\vec{H} \cdot \nabla \times \vec{E}_0^* = -j\omega_0\mu_0 \vec{H}_0^* \cdot \vec{H} \quad . \tag{3-6}$$

The addition of (3-5) and (3-6), and use of the identity

$$\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot \nabla \times \vec{A} - \vec{A} \cdot \nabla \times \vec{B}, \tag{3-7}$$

results in

$$\nabla \cdot (\vec{H} \times \vec{E}_0^*) = j\omega (\underline{\xi} + \Delta\underline{\xi}) \vec{E} \cdot \vec{E}_0^* - j\omega_0\mu_0 \vec{H} \cdot \vec{H}_0^* \quad . \tag{3-8}$$

The manipulation of (3-2) and (3-3) in a similar manner gives

$$\vec{E} \cdot \nabla \times \vec{H}_0^* = -j\omega_0 \underline{\xi} \vec{E}_0^* \cdot \vec{E} \tag{3-9}$$

and

$$-\vec{H}_0^* \cdot \nabla \times \vec{E} = j\omega\mu_0 \vec{H} \cdot \vec{H}_0^* \quad . \tag{3-10}$$

After adding (3-9) and (3-10), and using (3-7), one gets the analogous relation to (3-8):

$$\nabla \cdot (\vec{H}_0^{\,*} \times \vec{E}) = j\omega \mu_0 \vec{H} \cdot \vec{H}_0^{\,*} - j\omega_0 \underline{\xi} \vec{E}_0^{\,*} \cdot \vec{E} \,. \tag{3-11}$$

Now, add (3-8) to (3-11) and integrate over a closed volume τ

$$\iiint_\tau \nabla \cdot (\vec{H} \times \vec{E}_0^{\,*})\, d\tau + \iiint_\tau \nabla \cdot (\vec{H}_0^{\,*} \times E)\, d\tau$$

$$= \iiint_\tau \left\{ [j\omega(\underline{\xi} + \Delta \underline{\xi}) - j\omega_0 \underline{\xi}] \vec{E}_0^{\,*} \cdot \vec{E} + j\, \mu_0 (\omega - \omega_0) \vec{H} \cdot \vec{H}_0^{\,*} \right\} d\tau. \tag{3-12}$$

The divergence theorem states that

$$\iiint_\tau \operatorname{div} A \, d\tau = \iint_S \vec{n} \cdot \vec{A} \, dS \;, \tag{3-13}$$

where dS is an elemental area on the cavity surface and $\vec{n}$ is a unit vector extending out from the cavity surface at right angles to it. Therefore,

$$\iiint_\tau \nabla \cdot (\vec{H} \times \vec{E}_0^{\,*})\, d\tau = \iint_S \vec{n} \cdot (\vec{H} \times \vec{E}_0^{\,*})\, dS \tag{3-14}$$

and

$$\iiint_\tau \nabla \cdot (\vec{H}_0^{\,*} \times \vec{E})\, d\tau = \iint_S \vec{n} \cdot (\vec{H}_0^{\,*} \times \vec{E})\, dS \,. \tag{3-15}$$

With the identity

$$\vec{n} \cdot (\vec{B} \times \vec{C}) = \vec{C} \cdot (\vec{n} \times \vec{B}) \,, \tag{3-16}$$

and since

$$\vec{n} \times \vec{E}_0 = 0 \quad \text{and} \quad \vec{n} \times \vec{E}_0^{\,*} = 0$$

on the perfectly-conducting surface S, it follows that

$$\iiint_\tau \left\{[\omega-\omega_0][\underline{\xi}\vec{E}_0^{*}\cdot\vec{E}+\mu_0\vec{H}\cdot\vec{H}_0^{*}]+\omega\Delta\underline{\xi}\vec{E}_0^{*}\cdot\vec{E}\right\}d\tau = 0. \qquad (3\text{-}17)$$

Therefore,

$$\frac{\omega-\omega_0}{\omega} = -\frac{\iiint_\tau \Delta\underline{\xi}\vec{E}_0^{*}\cdot\vec{E}\;d\tau}{\iiint_\tau(\underline{\xi}\vec{E}_0^{*}\cdot\vec{E}+\mu_0\vec{H}\cdot\vec{H}_0^{*})\;d\tau}\;. \qquad (3\text{-}18)$$

For small perturbations $\vec{E} \doteq \vec{E}_0$, $\vec{H} \doteq \vec{H}_0$, and $\omega \doteq \omega_0$ so that

$$\frac{\omega-\omega_0}{\omega_0} = -\frac{\iiint_\tau \Delta\underline{\xi}\,|E_0|^2\,d\tau}{\iiint_\tau(\underline{\xi}|E_0|^2+\mu_0|H_0|^2)\;d\tau}\;. \qquad (3\text{-}19)$$

In the physical layout of the idealized experiment (Fig. 3-4), the plasma (region I) is enclosed by a glass cylinder (region II) surrounded by air (region III) which represents the space between the outside of the discharge tube and the cavity itself. In the actual setup, the tube protrudes well beyond the cavity, and the "dielectric-post" which was used to represent the plasma is not totally enclosed by the cavity. As a result, the analysis used here is imperfect and does not take into account fringing fields at the hole and the fact that the plasma is not totally enclosed by a perfect conductor.

If cavity-wall losses, other losses, and the presence of the glass containing the plasma represent all of the contributions to the resonant frequency, the resonant frequency ω_0 may be expressed as

$$\omega_0 = \omega_{0r} \left(1 + \frac{j}{2Q_0}\right), \tag{3-20}$$

where Q_0 is the Q of the cavity prior to the introduction of the plasma. Now, referring to (3-19), $\Delta \underline{\xi} = 0$ everywhere except in region I when the discharge creates the plasma. When the plasma is present, a new resonant frequency $\omega = \omega_r + j\omega_i$ is observed according to (3-19), with the numerator evaluated only over region I. For the TE_{10p} mode

$$|E_0|^2 = \frac{\omega^2 \mu_0{}^2 C^2 a^2}{\pi^2} \sin^2 \frac{\pi x}{a} \sin^2 \frac{p\pi z}{d} \tag{3-21}$$

and

$$|H_0|^2 = C^2 p^2 \frac{a^2}{d^2} \sin^2 \frac{\pi x}{a} \cos^2 \frac{p\pi z}{d} + C^2 \cos^2 \frac{\pi x}{a} \sin^2 \frac{p\pi z}{d}, \tag{3-22}$$

where C is a constant. If it is assumed that $|E_0|^2$ is constant over the region where $\Delta \underline{\xi} \neq 0$ (based on the fact that the odd-numbered modes have wavelengths large enough so that the fields at the center of the cavity are relatively constant, a reasonable approximation up to at least the $TE_{10\,11}$ mode), (3-19) becomes

$$\frac{\omega - \omega_0}{\omega_0} = -2 \frac{\iiint_\tau \Delta \underline{\xi}\, d\tau}{\epsilon_0 \, abd}, \tag{3-23}$$

where ϵ_0 is the permittivity of free space (note that the cavity shift is independent of p). Considering the plasma to have a complex conductivity

$$\underline{\sigma} = \frac{N_e e^2}{m(\nu_c + j\omega)} = \frac{N_e e^2}{m} \frac{\nu_c}{\nu_c^2 + \omega^2} - j \frac{N_e e^2}{m} \frac{\omega}{\nu_c^2 + \omega^2} , \tag{3-24}$$

and that, originally, air with a complex dielectric constant $\underline{\xi} = \epsilon_0$ occupied region I,

$$\frac{\omega - \omega_0}{\omega_0} = \frac{2}{\epsilon_0 abd} \left[\frac{e^2}{m(\nu_c^2 + \omega^2)} \iiint_\tau N_e d\tau + j \frac{\frac{e^2}{m} \frac{\nu_c}{\omega}}{\nu_c^2 + \omega^2} \iiint_\tau N_e d\tau, \right] \tag{3-25}$$

where e is the electronic charge, m is the mass of an electron, N_e is the number density of electrons, ω is the angular frequency of the electromagnetic wave, and ν_c is the collision frequency.

If the positive column of the discharge is controlled by ambipolar diffusion, the variation of the electron density in the radial direction of the tube (Fig. 3-5) is given by

$$N_e = N_{ec} J_0 (2.405 \frac{r}{R}), \tag{3-26}$$

where N_{ec} is the electron density at the axis of the tube, R is the tube's inside radius, and J_0 is the zero-order Bessel function with argument $2.405 \frac{r}{R}$. This electron density profile is derived on the basis of a model for the positive column in which positive ions are drawn to the wall of the discharge tube under the influence of a field produced by rapid radial electron diffusion; at the wall, the ions are neutralized [2]. If, on the other hand, charges are removed from the positive column entirely by volume recombination, then it is appropriate to assume that the electron density profile is constant over the cross section of the tube [3]. In practice, the charge balance in the positive columns of laboratory glow and arc discharges depends on some combination of ambipolar diffusion and volume recombination which is difficult to predict, especially in the hot-cathode discharges involved in the experiments reported in this research. Recently, however, Parker [4] calculated electron density profiles in cylindrical plasma columns using the model of Tonks and Langmuir [5]. In this model, a Maxwellian electron gas is contained by the radial electric field created when high speed electrons attempt to flow to the walls more rapidly than the massive ions can follow. The ions are assumed to be generated with negligible velocity at a rate proportional to the local electron density, and to fall directly to the walls under the influence of the radial electric field. This implies a negligible ion-neutral collision frequency so that the ions suffer negligible deflection as they move to the walls. Using the boundary condition that the current of ions and electrons to the insulating wall shall be equal, Parker derived an expression for electron density profiles in any gas, and a relation for the average electron density $\bar{N}_e$ divided by the electron density at the axis of the cylinder N_{ec} as a function of a parameter $\beta^2 = r_w{}^2/\lambda_{dc}{}^2 = r_w{}^2 N_{ec}/\epsilon_0 kT$, where r_w is the radius to the wall of the enclosed column, $\lambda_{dc}{}^2$ is the Debye length at the axis of the column, k is Boltzmann's constant, ϵ_0 is the permittivity of free space, and T is electron temperature. On the basis of the work by Parker and typical values of N_{ec}, λ_{dc}, and T which were either measured or estimated in the hot-cathode discharge, $\overline{N}_e/N_{ec}$ was calculated to be approximately 0.6. This value compares closely with the average electron density resulting from the profile of (3-26).

The only way that the electron density profile can be established accurately for the hot-cathode discharges of the experiments reported here is through direct measurement. Data taken by Cooper [6] with movable probes in discharge tubes whose dimensions were similar to the hot-cathode tube, and which drew equivalent currents, showed fairly close adherence to a Bessel function distribution, but his discharges were of the cold-cathode variety. On the other hand, analogous measurements with fixed Langmuir probes in a cold-cathode discharge with <u>exactly</u> the same dimensions as the hot-cathode tube indicate a departure from the Bessel function distribution and a close correlation to Parker's calculations. At best, the choice of (3-26) for an electron density profile is somewhat arbitrary; but, on the basis that it comes close to confirming the prediction for average electron density using Parker's results, and for the sake of convenience, it will be used in this analysis. Therefore,

$$\iiint_{\tau} N_e \, d\tau = \int_0^b \int_0^{2\pi} \int_0^R N_{ec} J_0\left(2.405 \frac{r}{R}\right) r \, dr \, d\theta \, dz = \frac{2\pi b N_{ec} R^2}{2.405} J_1(2.405) \quad (3\text{-}27)$$

and

$$\frac{\omega - \omega_0}{\omega_0} = K \left[\frac{\omega_{pc}^2}{\nu_c^2 + \omega^2} + j \frac{\nu_c}{\omega} \frac{\omega_{pc}^2}{\nu_c^2 + \omega^2} \right] , \tag{3-28}$$

where ω_{pc} is the angular plasma frequency at the center of the discharge $\left(\omega_{pc} = 2\pi f_{pc} = \sqrt{\frac{N_{ec} e^2}{m \epsilon_0}} \right)$, and K is a constant equal to $\frac{4\pi R^2 J_1(2.405)}{ad\,(2.405)}$.

With (3.20) and the new resonant frequency, it follows that

$$\frac{(\omega_r - \omega_{0r}) + j\left(\frac{\omega_r}{2Q} - \frac{\omega_{0r}}{2Q_0}\right)}{\omega_{0r}} \doteq K \left[\frac{\omega_{pc}^2}{\nu_c^2 + \omega^2} + j \frac{\nu_c}{\omega} \frac{\omega_{pc}^2}{\nu_c^2 + \omega^2} \right] . \tag{3-29}$$

The separate equations for the real and imaginary parts are

$$\frac{\omega_r - \omega_{0r}}{\omega_{0r}} = K \frac{\omega_{pc}^2}{\nu_c^2 + \omega_{0r}^2} \tag{3-30}$$

and

$$\frac{\omega_r}{\omega_{0r}} \frac{1}{2Q} - \frac{1}{2Q_0} = K \frac{\nu_c}{\omega_{0r}} \frac{\omega_{pc}^2}{\nu_c^2 + \omega_0^2} . \tag{3-31}$$

Solving (3-30) and (3-31) for the plasma frequency at the center of the tube, and the collision frequency, one gets

$$f_{pc} = \sqrt{\frac{f_{0r}}{K} (f_r - f_{0r}) \left[\left(\frac{\nu_c}{\omega_{0r}} \right)^2 + 1 \right]} \tag{3-32}$$

and

$$\nu_c = \frac{\pi f_{0r}^2}{f_r - f_{0r}} \left[\frac{f_r}{f_{0r}} \frac{1}{Q} - \frac{1}{Q_0} \right] . \tag{3-33}$$

Because Eqs. (3-32) and (3-33) apply only to a "plasma dielectric post" whose ends terminate in the walls of a totally closed cavity, possible correction factors were considered for the plasma and collision frequencies which would take into account the field configurations at the open hole through which the discharge tube protrudes. In an experimental step to determine the nature of such adjustments, various dielectric materials with known dielectric constants and loss tangents were enclosed by a cylindrical glass tube whose diameter and wall thickness were equal to those of the actual discharge tube. Each of these assemblies was long enough to simulate the discharge tube, and was positioned in the cavity hole in the same manner for measurements of Q and the shift from the resonance frequency of the glass alone. Liquids that were used for this calibration included Aroclor 1232* and Aroclor 1242* (high loss tangents), carbon tetrachloride and heptane; the solids were polystyrene, Micarta 496, and woods such as balsa, poplar, and fir. The liquids were poured into cylindrical glass containers whose openings were sufficiently far away from the cavity hole to have no effect on the fields at the hole. The solid dielectrics were cylindrical rods, about seven inches long, machined so that their diameters were very close to the inside diameter of an open-ended glass tube. Thus, they could be easily inserted or withdrawn when necessary. All dielectrics extended three inches or more from either

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*Monsanto Chemical Company

wall of the cavity and terminated at a point where negligibly small fringing fields were observed.

With the perturbation formula used to derive (3-32) and (3-33) [Eq. (3-23), p. 3-8], and known values of dielectric constant and conductivity [9] for each material at the RF used for the calibration,* theoretical resonant frequency shifts and Q's were computed and compared to measured values. As shown in Table 3-2, the measured shifts in resonant frequency consistently required a multiplicative factor of approximately 2.50 (average) to bring them into line with the theoretical values,† whereas the measured Q's required almost no correction. On this basis, measured $(f_r - f_{or})$ would have to be multiplied by 2.5 in Eqs. (3-32) and (3-33) when the latter relations are used to determine plasma frequency and collision frequency by measuring the resonant frequency and Q of the cavity both before and after the introduction of the plasma. However, the discussion which follows indicates an additional correction which would tend to nullify this multiplication of the measured frequency shift.

Several workers in the past have used a diagnostic technique for plasma posts in a configuration equivalent to that used here [10-13]. However, they failed to consider important plasma polarization phenomena which, if not accounted for, can result in serious overestimates of plasma frequencies and collision frequencies. Using empirical methods, Leiby [14]

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*Approximately the same RF's as those used in the actual plasma measurement.

†Such a factor was unecessary in the application of Eq. (3-23) to measurements of dielectric constants with a completely closed cavity. The general perturbation formulas derived in 3.2.2 were used to find the dielectric constant for small slugs of various solid dielectrics inserted as centered posts in the closed cavity, and were found to yield values very close to the accepted standards for the materials involved [9].

Sample	Resonant Frequency Mc/s	Bandwidth Mc/s	Measured Resonant Frequency Shift (Referred to Glass Pipe) Mc/s	Standard Value of Dielectric Constant (von Hipple[9])	Theoretical Q Based on Standard Value of Conductivity (von Hipple[9])	Resonant Frequency Shift Required to Give Standard Dielectric Constant Mc/s	Multiplicative Correction to Measured Resonant Frequency Shift	Measured Q	$\frac{Q_{standard}}{Q_{measured}}$
None	891.175	0.466	-	-	-	-	-	1918	-
Cylindrical Glass Pipe	887.400	0.550	-	-	-	-	-	1614	-
Balsa	886.596	0.820	0.804	1.25	1090	2.08	2.59	1080	1.01
Poplar	885.590	1.165	1.810	1.55	853	4.66	2.58	760	1.14
Fir	884.350	1.355	3.050	1.84	606	7.14	2.37	652	0.93
Polystyrene	882.220	0.675	5.185	2.55	1570	12.69	2.45	1310	1.20
Micarta 496	878.215	4.175	9.190	3.89	223	22.45	2.45	210	1.06
Aroclor 1242	881.365	4.840	6.040	2.82	166	15.42	2.56	182	0.91
Aroclor 1232	881.010	10.360	6.390	3.05	81.2	17.31	2.71	85	0.96
Heptane	884.105	0.740	3.295	1.97	1590	8.24	2.50	1195	1.33
Carbon Tetrachloride	883.110	0.605	4.290	2.17	1604	9.92	2.31	1460	1.10
						Average	2.50		1.07

Table 3-2. Cavity calibration with various dielectric materials.

has investigated these polarization effects by making successive independent measurements of the same discharge tube in a cylindrical cavity and in a waveguide. In this manner, he showed the necessity of modifying the complex dielectric constant of a plasma post.

Leiby's RF measurement of dielectric constants in afterglow plasmas confirmed the existence of polarization effects, and the variation of plasma and waveguide dimensions revealed that the dielectric constants were geometry dependent. Leiby concluded that the probing RF in the plasma-hole system was measuring an "effective" dielectric constant which was based upon a description for the plasma in terms of two separate polarization fields. In addition to the polarization arising from electron inertial effects in such a model, Leiby postulated surface polarization charges to account for the fringing of the incident microwave electric fields at the holes where the plasma enters the cavity. Thus, the effective dielectric constant gives a correct description of the ionized gas for fields external to a plasma by considering the combined effect of polarization fields in a plasma in the absence of fringing, plus a polarization due to other charges on the cavity walls near the hole. This picture is similar to one that can be envisioned in a parallel-plate capacitor in which the charges on the outside of the plates near the edges (which are the sources of the fringing fields) are combined with those on the inside surfaces so they may contribute to the polarization of the dielectric. If the dielectric constant for the medium between the plates were described in terms of polarization fields with no fringing, the capacitance calculated on this basis would not agree with the measured capacitance of the system. By modifying the calculated polarization with an additional "polarization" resulting from the extra charges, a total polarization could be determined which would yield a capacitance equal to the measured value. As a final step, the dielectric could be described with another polarization field which would neglect fringing by making the new polarization equivalent to the previously determined total polarization. It is true that this procedure does not result in a true description of fields internal to the plasma; however, for measurement purposes, only fields external to the plasma are important.

In the large majority of cavity diagnostic measurements, circular cylindrical cavities are used which operate with modes that have only θ-directed electric fields; thus, no surface polarization charges are induced. The absence of this complication for cylindrical cavities probably accounts for the fact that the polarization effect has not received any great attention until recently.

The effect of the surface-charge polarization field is to alter the complex dielectric constant of the plasma so that (1-2) becomes

$$\underline{\xi} = 1 - \frac{\omega_p^2}{\omega^2 - \mathcal{L}\omega_p^2 - j\nu_c\omega} . \tag{3-34}$$

Here $\mathcal{L}$ is a geometrical depolarization constant which is a measure of the amount of fringing at the holes in the cavity or waveguide [14, p. II-15]. $\mathcal{L}$ depends solely on the dimensions of the hole and the discharge tube and can have a value anywhere between zero and one. In Leiby's experiments, discharge-tube diameters were 0.28 cm and 0.45 cm for which $\mathcal{L}$ was 0.036 and 0.074, respectively. These diameters are much less than the diameter of the discharge tube in the present research, but Leiby's work indicates that $\mathcal{L}$ increases with increasing diameter.

When Eq. (3-34) for the dielectric constant of the plasma is used in Eq. (3-23), and expressions are derived for the plasma and collision frequencies in terms of the measured resonant frequency shift and Q, it is found that the frequency shift must be corrected by a varying multiplicative factor which is less than one and that depends upon $\mathcal{L}$ and the plasma frequency at which the measurement is made. In addition, the collision frequency determined from Eq. (3-33) has to be multiplied by a term $(1 - \mathcal{L} \frac{\omega_p^2}{\omega^2})$ to account for the surface polarization phenomena. Failure to make such a correction in the collision frequency could make the latter appear dependent upon the discharge current as shown in the lower half of Fig. 3-6 (a). For example, with a discharge current which puts the plasma frequency somewhere near the RF, and an $\mathcal{L} = 0.5$,* one could wrongly conclude that the collision frequency was about twice as large as the true value. Of course, the degree of error is a function of the plasma

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*Leiby showed that $\mathcal{L}$ was proportional to $(d/b)^{3/2}$, where d and b are the plasma diameter and minor dimension of the waveguide, respectively. Using Leiby's empirically determined $\mathcal{L}$ values for a 0.28 cm diameter discharge tube in L-band waveguide as a basis, $\mathcal{L}$ for the configuration used in this research was estimated to be in the vicinity of 0.9.

frequency for fixed RF and increases with the discharge current. Furthermore, the exact value of the correction depends on $\mathcal{L}$ (which would have to be determined separately). A convenient way of doing this would be to pulse the discharge tube and make simultaneous afterglow measurements of dielectric constant in the plasma-post configuration, as well as with a circular cylindrical cavity. By comparing the two over a broad range of plasma frequencies in the afterglow, as Leiby did, $\mathcal{L}$ could be deduced.

Figure 3-6 shows typical results of plasma frequency and collision frequency measurements for actual situations encountered with the discharge tube used in this research. Here, $R = 58.5/2$ mm, $A = 0.3048$m, and $d = 1.832$m, so that $K = 4.16 \times 10^{-3}$. In general, the plasma frequency was related to the discharge current through an expression of the form $f_{pc} = c I^{m}$, where c was a constant and m varied between 0.50 and 0.65. However, data on the collision frequency were not always reliable and reproducible. Part of the difficulty was due to the inherent errors in trying to measure the Q of the cavity for low collision frequencies, where the small loss resulted in bandwidths that were too narrow to resolve accurately with the frequency counter in the 500 to 1000 Mc/s range used in the experiments. At higher discharge currents, the measurements were far more accurate. However, at these values it becomes difficult to find the resonant frequency, so that the data for the plasma frequency have a greater error than in measurements at lower discharge currents. An error analysis, taking into account only the measurement error contributed by instrument limitations (and not the validity of the perturbation theory

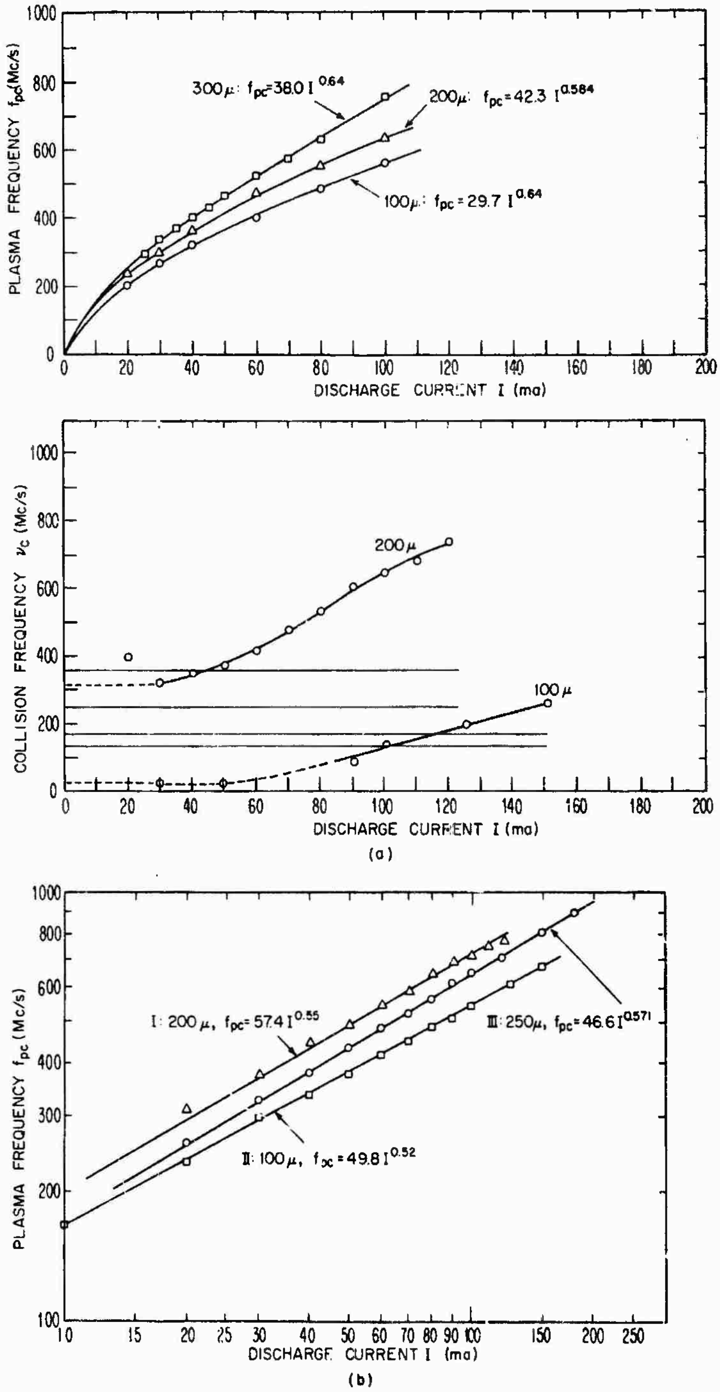

Fig. 3-6. Measurements of plasma and collision frequencies for various gas pressures in positive column of neon dc discharge using resonance shift method in rectangular cavity. (a) Linear coordinates. f_{or} corresponding to TE_{109} mode (≈ 887 Mc/s). (b) Log-log coordinates. I: f_{or} = 887.890 Mc/s, Q_o = 1520. II: f_{or} = 887.795 Mc/s, Q_o = 1755. III: f_{or} = 1028.585 Mc/s, Q_o = 1590. Relations between f_{pc} and discharge current I are determined empirically from each resultant curve.

applied to this situation) shows that, for higher currents, collision frequency is in error by at most 30 %, while the plasma frequency is in error by no more than about 14%. For very low currents, the collision frequency can be in error by as much as 110%, while the plasma frequency is probably accurate to within 5%. These measurement errors are in addition to those introduced by failing to correct the observed resonant frequency shift with the multiplicative factor 2.5, and by not considering the surface-charge polarization effect.

Figure 3-6 (a) is typical for the measurements of collision frequency. These data have not been corrected for the surface-polarization effect.* As discussed previously, this phenomenon represents a possible explanation for the departure of the collision frequency from a constant value which it should have at a fixed pressure regardless of the discharge current. Because of the difficulty in measuring the Q for low currents, the data in each curve at the bottom of Fig. 3-6(a) do not conclusively show a constant collision frequency in the range of currents for which a negligible correction is required. To allow for the possible measurement error, the 0.2 Torr curve is dotted below 30 ma. In the 0.1 Torr case, where the collision frequency is very low, a larger range is in doubt. However, both curves

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*Based on the data of Fig. 3-6(a), $\mathcal{L}$ would have to have a value of approximately 0.8 to make the collision frequency independent of discharge current. This value compares with $\mathcal{L} \approx 0.9$ as previously estimated from an extrapolation of Leiby's results to the geometry used in this research.

indicate a definite flattening with decreasing plasma frequency (current), and are consistent with the nature of the correction at higher currents.

As a check on the magnitude of the collision frequencies indicated by the flat portion of each curve, theoretical values were computed using the relation $\nu_c = p_o P_c v_e$, where p_o is the pressure of the discharge in Torr reduced to room temperature T ($p_o = p \frac{273}{T}$), P_c is the probability of collision between an electron and neutral molecule, and v_e is the average electron velocity in cm/sec, related to the electron temperature T_e through the relation $\frac{1}{2} m v_e^2 = \frac{3}{2} kT_e$, where m is the mass of the electron and k is Boltzmann's constant [15]. T_e was determined in each case using the theory of the positive column in a glow discharge [16, pp. 239-240]. For the 0.2 Torr case, ν_c was calculated at between 250 Mc/s and 350 Mc/s depending on the value one takes for P_c at the assumed temperature (theoretical and measured data for P_c as a function of electron temperature show considerable lack of agreement [15, pp. 7, 16]). These limits, shown by the broken horizontal lines in Fig. 3-6(a), bound the measured data nicely in the range of negligible correction. A range of 135 Mc/s to 170 Mc/s was calculated for a 0.1 Torr discharge, and these limits are well above the measured data at low discharge currents. This is probably due to the inability to accurately read the pressure in the discharge tube with the gauge that was used in the experiment. Based upon previous experience with the calibration of the pressure gauge, it would have been quite possible to overestimate the pressure by as much as 25%. If the pressure in the tube were only 0.075 Torr, for example, the calculated range of collision frequency would have bounded the measured data.

Collision frequency in a discharge tube at fixed pressure should only vary with discharge current if the electron temperature itself is not a constant. Such a situation can occur in a discharge tube as a result of departures from a Maxwellian electron velocity distribution due to small changes in the electric field with discharge current, although such perturbations are not common in the type of discharge that was used here. Even if they did occur, they would cause variations in collision frequency of less than 10%. However, just to make sure that the electron temperature remained essentially constant, Langmuir probe measurements of this quantity were made in another discharge tube with the same dimensions as the one used for the measurements of Fig. 3-6 (a), and under similar operating conditions. As expected, the electron temperature did not change for any currents in the 25ma to 150 ma range.

Large discrepancies in the collision-frequency data can also stem from misleading measurements of cavity Q due to microwave propagating modes along the plasma column external to the cavity. The ultimate radiation of such RF power would lower the observed Q from the value it would have had from the effect of the plasma alone. However, these modes cannot occur at electron densities less than about 5×10^{10} electrons/cc in the cavity plasma-post configuration [14, 17]. Since such densities are well above the maximum densities generated by the highest discharge currents encountered in these experiments, this explanation for variations in collision frequency does not seem pertinent in explaining the apparent variation of collision frequency with discharge current in Fig. 3-6(a).

It must be emphasized that the preceding discussion of possible correction factors in the plasma-frequency and collision-frequency measurements does not preclude the possibility of either additional or alternative approaches to the interpretation of the data. Although the considerations presented appear to offer consistent explanations for discrepancies in the cavity plasma-post diagnostic technique, especially with regard to the collision-frequency data, the validity and application of this measurement method has been under investigation for some time [18], and is still not fully resolved. Leiby's results, however, seem to be pertinent to the measurements reported in this research. In addition, Johnson has looked into the theoretical aspects of the problem recently [19], and claims to have deduced a formal reason for the introduction of Leiby's empirically derived modified plasma dielectric constant. Because these developments seem to have bearing on the present measurements, and appear to have a reasonable basis, they have been considered in some detail. Again, however, any final conclusions on the procedural error must depend upon the results of further investigations.

3.3 Discharge Tubes

Details of the discharge tube used for the measurements appear in Fig. 3-7. Corning Pyrex brand 7740 borosilicate glass is utilized throughout, except for the Kovar-to-glass seal at the end of the transducer in which Corning 7052 glass is used, and the Corning 3320 glass for sealing the tungsten rod in the anode extension. Wall thicknesses for the diameters involved in the 7740 glass are roughly 2.25mm. Its other characteristics include [7]:

Strain Point:	515°C
Annealing Point:	565°C
Softening Point:	820°C
Working Point:	1245°C

All metallic parts within the discharge tube are made from nickel, tungsten, or Kovar. The nickel anode is spot-welded perpendicular to the axis of a nickel tube with a 0.1875" outside diameter and a 0.015" wall. Forced over the outside of this tube is a Kovar slug, machined for a slide fit within 0.491" I.D. glass tubing. This assembly, in turn, slides over a tungsten rod which protrudes through the end of the discharge tube and provides the electrical connection to the anode. By means of a heavy permanent magnet whose field couples to the Kovar slug, the anode can be conveniently positioned from outside the tube.

Raytheon 5586 cathodes used in these experiments have a barium carbonate and strontium carbonate coating embedded in a nickel wire mesh which is wrapped around a cylindrical support. Other than the filament which is made of tungsten, the remaining support structure and electrical

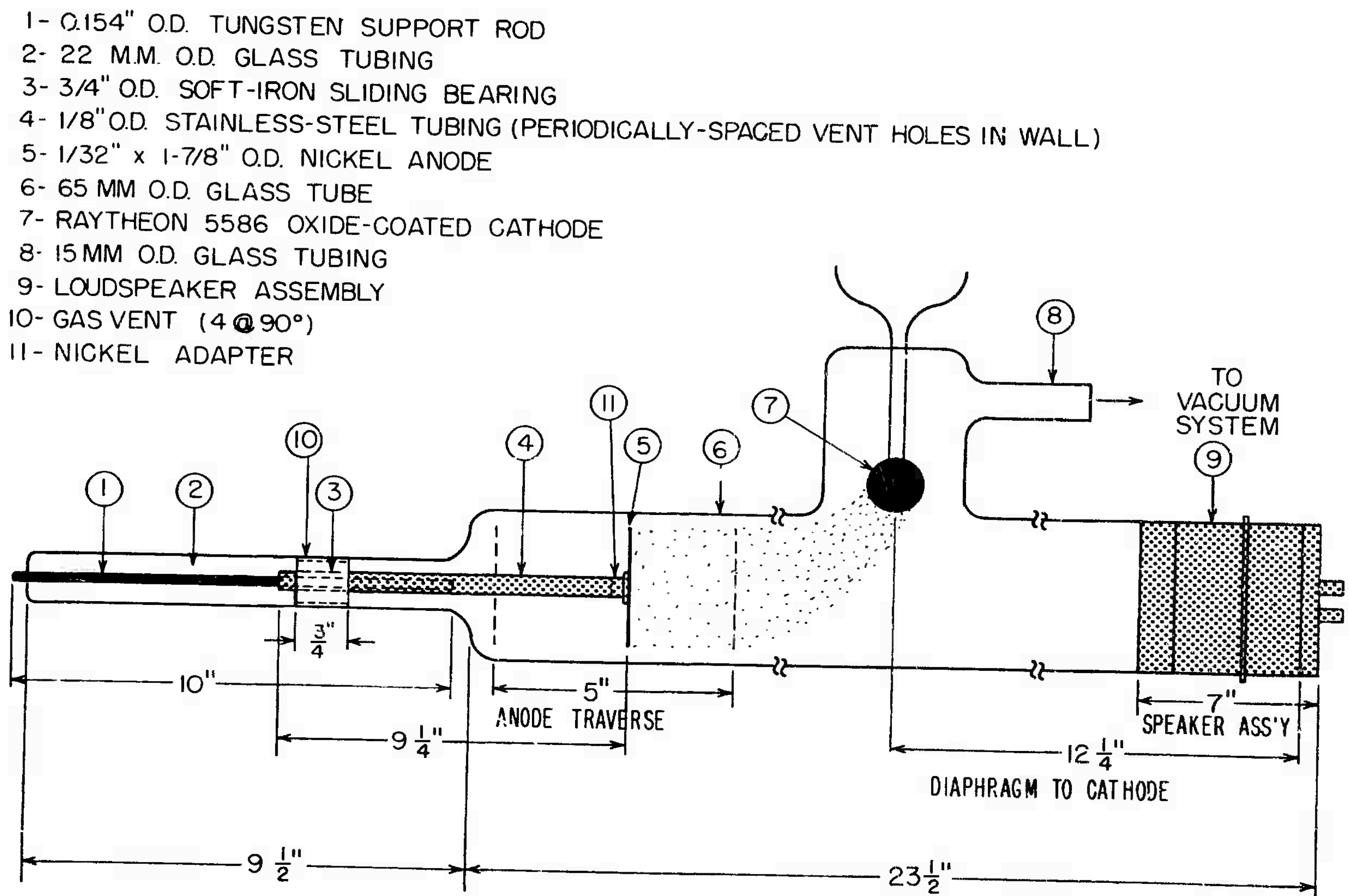

Fig. 3-7. Cylindrical discharge tube incorporating transducer and moveable anode.

leads are made of nickel and high alumina ceramic. When converted, the cathodes give off carbon dioxide leaving oxides of barium and strontium which serve as the emitting surface. Both of these elements are highly unstable; exposure to water vapor or air causes a chemical reaction which breaks them down and produces hydroxides which are physically weak, tend to chip, and — more important — do not emit electrons efficiently. As a result, once a cathode is converted, one must be very careful not to allow air to enter the discharge tube in appreciable quantities. The presence of inert gases such as neon or argon has a negligible effect on the cathode.

Once the cathode has been installed, a complete conversion can be made within 30 minutes. The procedure consists of raising the heater voltage gradually up to about 25 v dc corresponding to a temperature of about $1000^{\circ}C$. Conversion may be monitored on both a thermocouple gauge and an ionization gauge, although the latter should be inactive until the process is almost complete. At about 12 v dc, the binder is driven off resulting in a quick jump in pressure which can be observed on the thermocouple gauge, whereupon the pressure drops to its original value. When the heater voltage reaches about 20 v dc, the conversion reaches its peak accompanied by large pressure rises (as high as 0.5 Torr) for periods of one or two minutes. At this point, the mechanical pump is doing most of the pumping and, once it is able to absorb the large amounts of carbon dioxide evolved from the cathode, the pressure begins to drop and the diffusion pump takes over again. Then the voltage is raised to a maximum of 25 v, the ionization gauge turned on, and the pressure allowed to drop at least to the low end of the 10^{-5} Torr scale. Normal operating voltage for the cathode is between 16 and 18 v dc corresponding to temperatures of $700^{\circ}C$ to $900^{\circ}C$.

Because reasonably high heater voltages are used both during the conversion process and in normal operation, dc sources are preferred to ac heater supplies, since the latter often create unwanted ionization, and subsequently, a discharge between the cathode emitting surface and the remainder of the cathode support structure. In fact, discharges have been observed in the dc case when the polarity of the power supply placed the support for one of the heater leads at a higher potential than the support for the other lead which, unfortunately, is connected to the cathode emitting surface. Such discharges are undesirable; they not only cause excessive currents and heating at the cathode surface, but can also result in sputtering of the barium and strontium.

Early discharge tubes used cold-cathode discharges instead of the type just described; in this case, a metallic disc similar to the anode was placed in the T-extension. Such a discharge requires a high-voltage power supply and usually draws small currents through the tube (corresponding to low electron densities). A typical cathode-anode voltage drop of about 300 v in a 1-Torr discharge drew an average current of no more than 40 or 50 ma. In practice, sources with greater output voltages are required since, in the cold-cathode case, ignition voltages are usually quite a bit higher than running voltages. For pure inert gases, such as neon, ignition voltages of 500 v or 600 v are frequently required, even when the discharge is initiated with a Tesla coil (the usual way to begin the ionization and breakdown process). Argon, with its lower excitation potential, is a little easier to ignite, and in fact small traces of this gas are added to neon to make neon discharge with lower-voltage power supplies.

Cathode sputtering (caused by ions bombarding the cathode surface under a high accelerating potential) is another disadvantage of this type of discharge since, ultimately, for a continuous dc discharge, large amounts of metal coat the inside walls of the tube. For the geometry involved here, this is not a serious problem since most of the measurements are made in the positive column far away from the cathode; but, there are situations where such sputtering is undesirable.

Hot-cathode discharges, on the other hand, make it possible to draw high currents with low tube-voltage drops and readily available dc power supplies. The main disadvantage in this case stems from the inability of the cathode to maintain a constant state of electron emission from one day to the next. Even if the cathode temperature were accurately controlled, the work function of the emitting surface would continually undergo gradual changes due to aging and chemical reactions, and slight variations in cathode emission can produce significant changes in the voltage and current characteristics of the tube. The degree of electron emission also seems to have a major effect in either enhancing or eliminating the natural low-frequency oscillations which occur in the discharge itself. In fact, a small flexibility in adjusting the cathode temperature frequently provides a convenient way of controlling the oscillations; however, temperature variations influence the operating conditions of the discharge on an irregular basis. In any event, cathode currents as high as 200 ma have been drawn through the discharge for anode-cathode voltages of between 40 and 50 v. According to data supplied by Raytheon, the cathode is capable of drawing currents close to one ampere if the driving circuitry could support them.

Cleaning of all parts within the discharge tube is essential in assuring reproducible experiments. With the exception of the cathode, all metallic parts are hydrogen fired and chemically cleaned prior to the assembly of the discharge tube, and chemically cleaned again after the tube is assembled. There are several chemical solutions which are used for cleaning purposes depending upon the materials involved,* but for our components a solution of 45 parts hydrochloric acid, 5 parts nitric acid, and 50 parts water, heated to 60°C, was sufficient. Parts to be cleaned were dipped in this solution for no more than 30 seconds, rinsed in boiling water, and dried in air after immersion in acetone. Because of the difficulty in applying this procedure to the parts once they were in position in the discharge tube, rinsing with acetone or carbon tetrachloride was considered sufficient after assembly. Once the discharge tube was sealed to the vacuum system, the anode was vacuum fired with an RF induction coil placed around the outside of the discharge tube in proximity to the anode itself. With the vacuum system pumping at full capacity, the anode temperature was raised

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*An excellent reference for processing of tube parts is the M. I. T. Tube Laboratory Manual [8].

to a point where the nickel was almost white hot, and held there until the pressure in the tube began its gradual drop to levels that existed prior to the heating.

For bake-out purposes, heating tapes are wrapped around all sections of the tube, including the transducer housing, and raised in temperature until the glass itself is at approximately 450°C. The tube is allowed to remain at this temperature for periods of up to 36 hours while the vacuum system pumps on the tube, although the actual bake-out time depends on the level to which the pressure within the tube drops. Bake out is usually done with the cold trap empty. When the pressure reaches midway on the 10^{-6} Torr scale, the tapes are removed, the glass is allowed to cool, the cold trap filled, and the pumping process continued. This tube-processing procedure has resulted in pressures as low as 7×10^{-8} Torr in the discharge tube, and has proved quite adequate in eliminating contamination within the system.

3.4 Instrumentation and Circuitry

Filtered output from a 500-v 200-ma dc power supply provides the excitation source for the discharge tube. A 500-ohm load resistor is in series with the tube along with a transformer which enables an oscilloscope and wave analyzer to be coupled into the tube circuit while maintaining connection to a common ground. The frequency response of this transformer has been checked independently and found to be "flat" within 0.5 dB from 20 c/s to about 20 kc/s. As a result, output from its secondary can be fed directly to an oscilloscope to monitor the low-frequency plasma oscillations when they occur in the tube current. However, the primary objective of this arrangement is to couple a low-frequency signal in the audio range into the discharge-tube circuit while simultaneously measuring the resultant plasma-modulation effect. In this manner one is able to calibrate the magnitude of plasma-frequency variations caused by the loudspeaker.

During normal operation, the filament supply for the discharge-tube cathode provides between 16 v and 18 v at approximately 3.2 a. This power supply, which was constructed in the Gordon McKay Laboratory, consists of four semiconductor diode rectifiers connected as a full-wave bridge, and filtered with a simple RC network. Some 120 c/s and higher harmonic signals resulting from the imperfect filtering do couple, in small amounts, into the discharge-tube circuit and have a slight effect on the discharge-tube current. As a result, one must be careful not to confuse apparent modulation of the plasma because of this effect with modulation produced by the transducer or plasma oscillations themselves.

Simplicity in the RF detection system of Fig. 3-1 is a result of the high sensitivity and selectivity, as well as the versatility, of the wave analyzer. When the sound source is activated, the plasma-modulated cavity output is crystal detected and fed directly to the analyzer which has a full scale sensitivity of 30 μv with 0.1-μv resolution over a frequency range from 20 c/s to 50 kc/s. For plasma- and collision-frequency measurements of the unperturbed medium using the cavity resonance-shift technique described in 3.2.2, the CW cavity output is fed to a crystal diode modulator, amplitude-modulated at 1000 c/s, detected, amplified, and fed to the wave-analyzer which is tuned to 1000 c/s. This scheme is extremely rapid and straightforward, and permits the taking of data even for very high plasma and collision frequencies when very small amounts of RF power appear at the cavity output. The usual procedure is to measure the resonant frequency and Q of the cavity without the plasma prior to the start of measurements, then to activate the discharge and make similar measurements for selected discharge currents.

Frequency of the RF input is carefully monitored with a Hewlett-Packard Model 524C frequency counter. This intrument has a frequency stability far beyond that of the RF driving oscillator, and an accuracy greater than that required in the measurements. For all practical purposes, the limiting factor in the frequency measurements is the oscillator itself which has a measured drift in the vicinity of 50 kc/s at close to 1000 Mc/s. This is certainly adequate for the data that have to be taken, and is less critical than the "settability" of the oscillator which creates a problem during the determination of the magnitude of the plasma-frequency modulation.

In addition to an automatic frequency-control circuit for tracking slow drifting signals, the wave analyzer has a provision to operate as a generator of very stable signals with magnitudes up to one volt. Operating as a beat-frequency oscillator (BFO) in this manner, the calibration signals may be coupled into the discharge-tube circuit and automatically tracked by feeding the crystal-detected output into the wave-analyzer input. In this way, the wave analyzer functions simultaneously both as a signal source and tuned voltmeter. The wave analyzer is also used to measure the spectrum of the low-frequency plasma oscillations observed across the 500-ohm resistor in the discharge-tube circuit, and as the analyzer of such oscillations when they are detected by the transducer operating as a microphone. In the latter case, the microphone output is amplified and connected to the wave analyzer.

Power levels of RF probing signals can often affect the plasma itself. Plasma heating and associated losses are the usual result, in addition to the creation of nonlinearities in the ionized medium. A check of this possibility was made by comparing measurements with maximum input power to the cavity with those at much reduced levels. Using all the available power from General Radio's 1209-B oscillator (about 300mw), there was no appreciable difference in the comparative measurements. This fact was established for the plasma-modulation data as well as for the measurements of resonant frequency and Q. However, as a general rule, power levels were kept to minimum values consistent with noise-free measurements.

3.5 Vacuum System

Designed and constructed for the preparation and cleaning of the discharge tube and its components, the vacuum system used in this work has provided ultimate pressures of 4×10^{-8} Torr for tube processing and has demonstrated the capacity to handle a fast and efficient coated-cathode conversion. For maximum experimental flexibility, the discharge tube is connected to the vacuum system at all times so that the plasma pressure may be monitored continuously and varied at will, the tube can be filled with various gases, and the system can pump on the tube when experiments are not in progress to provide continuous cleaning, thereby enhancing the probability of reproducible measurements through minimum contamination.

Referring to Fig. 3-8, the vacuum system uses a 3-stage water-cooled oil-diffusion pump with a maximum pumping speed of 44 liters per second at pressures below 9×10^{-4} Torr. Although the manufacturer's specifications call for the use of Octoil pump fluid, Octoil-S has been found to produce lower ultimate pressures than regular Octoil and has been used almost exclusively.

With the exception of the back-fill and leak-detection valve, all valves are of the brass block-bellows type with Teflon-seating-gaskets and O-rings.* Valves 7, 8, 9, and 10 in Fig. 3-8 are all one-inch valves while the back-fill valve which controls the intake of the discharge gas has a 1/2" port. The manifold, like the valves, is made of brass; brass components contrast to those of bakeable stainless-steel in ultra-high vacuum systems. However, the inability to drive contaminants out of the manifold and valves has not been a major hindrance since the ultimate pressures reached with the system are more than adequate for the purposes of the experiments.

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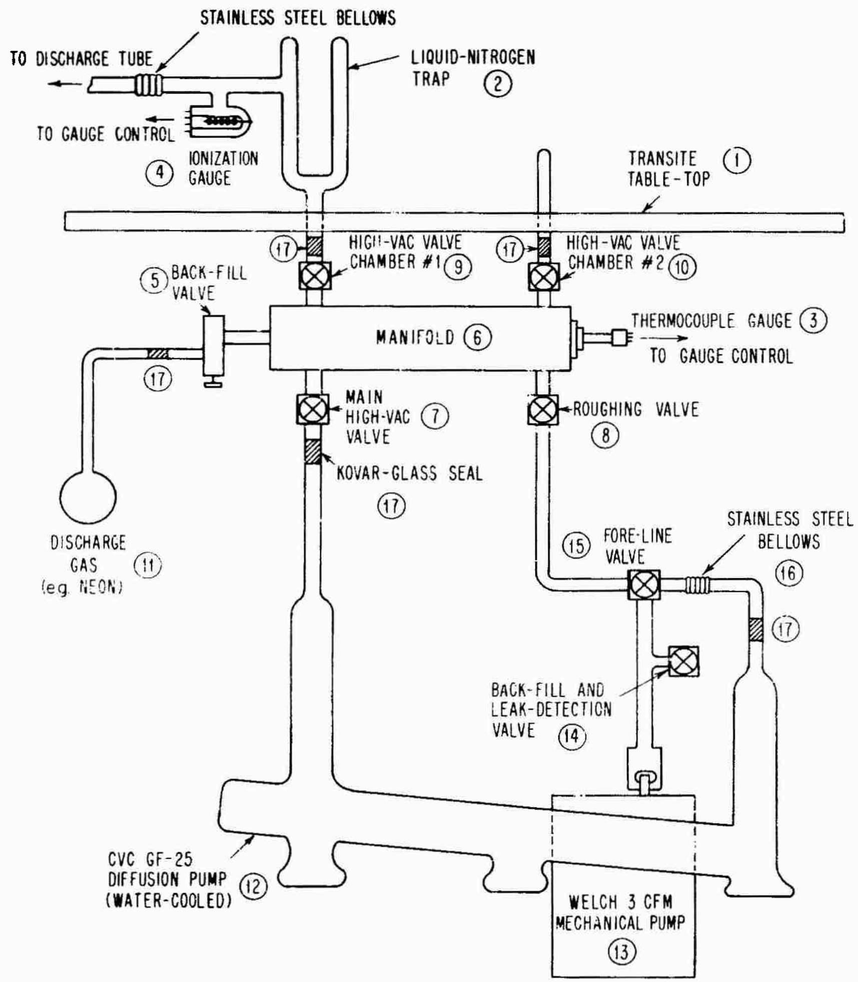
*Maximum leak rate measured close to the discharge tube with valve 9 closed is approximately 10^{-8} cc/sec.



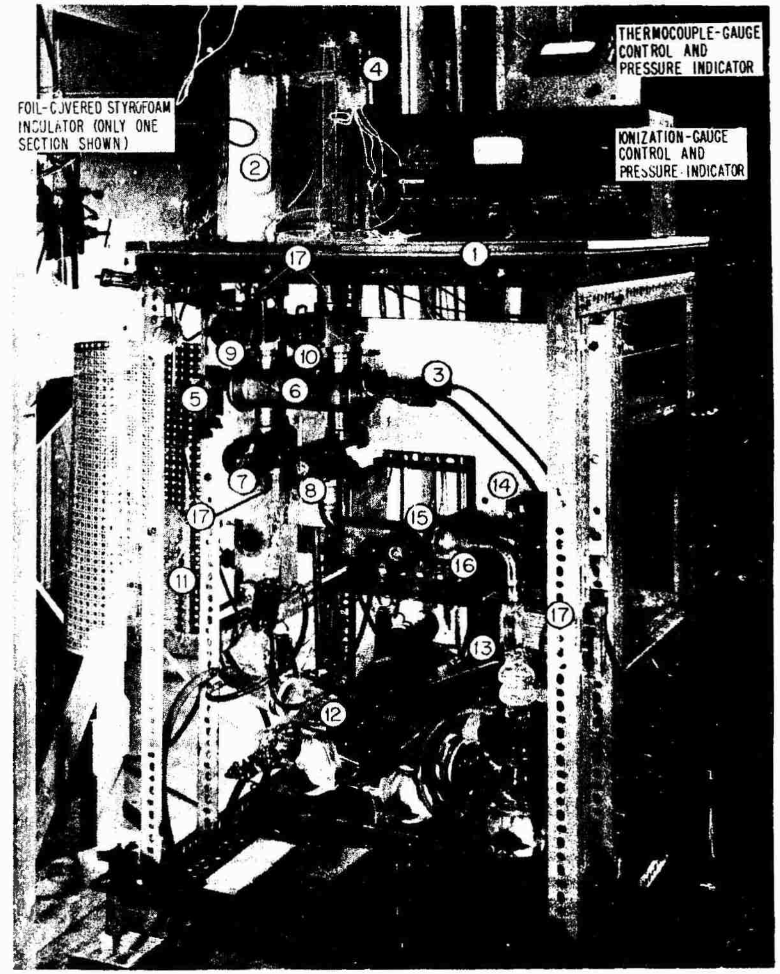


Fig. 3-8. Vacuum system for processing discharge tubes and converting hot cathodes.

Placement and construction of the liquid-nitrogen cold trap are novel features of the system. By placing the trap (2 in Fig. 3-8) above the transite table top, it is possible, prior to attachment of the discharge tube, to place a high-temperature oven over the top of the vacuum system to bake the cold trap, the gauge, and the tubulation to the load. Ordinarily, cold traps are not this accessible. But, in this case, it is important to clean the trap constantly since the carbon dioxide and other vapors evolved during the conversion process are apt to collect on its walls and be an ever-present source of contamination, especially in the absence of liquid nitrogen. When the cold trap is in operation, it is enclosed by a Styrofoam insulator covered with aluminum foil to minimize the evaporation of liquid nitrogen. The liquid nitrogen drains into the cold trap from a 25-liter dewar at a rate determined by an oxygen-activated level control which keeps the cold trap completely filled at all times.

High-vacuum pressures are observed and monitored by a Bon-De type BD-30 ionization gauge which has been found to be interchangeable with the less reliable Veeco RG-75 single-filament tube whose filaments are extremely fragile. The former works quite well with Veeco's RG-2A electronic controls and pressure indicator. For measuring higher pressures when the discharge tube is filled with the gas that creates the plasma, a Veeco SP-1 millimeter-range vacuum gauge is used. These pressures are measured in the manifold and, under the static conditions of back-fill with the main high-vac valve closed, closely approximate the pressures in the discharge tube itself. The operating range of this gauge is from 100 microns to 20 Torr, and it is calibrated over this range for the noble gases and hydrogen. Below 100 microns and above 20 Torr, pressure readings with this gauge are quite misleading and can result in serious errors.

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4. EXPERIMENTAL RESULTS

Initial measurements were made in stable and quiescent discharges over a limited range of pressures for which large spontaneous low-frequency oscillations were not observed. The transducer operated exclusively as a loudspeaker in the experimental configuration of Fig. 3-1 to produce a sizable disturbance in the positive column of the plasma and make it possible to detect variations in both the plasma frequency and collision frequency. Using the cavity frequency-response characteristics in conjunction with plasma- and collision-frequency data from separate cavity resonance and Q measurements, a calibration scheme was developed to determine the magnitude of the plasma-frequency perturbation.

Frequent and unpredictable appearance and disappearance of moving striations made it extremely difficult to maintain a quiet plasma during the early work. In fact, as measurements progressed there were indications that the plasma was often in a quasi-stable condition, at which time the sound waves, transmitted into the medium by the transducer, actually excited a low-frequency oscillation and interacted with it. This phenomenon was not entirely unexpected in view of the recent work by Cooper [1] at M. I. T. in which dc discharges were pulsed electrically to investigate the nature of traveling ionization waves in slightly ionized gases.

Ultimately, a separate investigation was conducted to determine the nature of the interaction between the large spontaneous plasma oscillations and the sound waves generated by the transducer. In addition to an attenuation of the sound waves propagating into the plasma, a clear-cut frequency mixing was observed between the low-frequency oscillations and the forced sound wave.

The transducer was also used as a microphone without the RF circuitry to "listen" to the sound waves in the neutral gas which were generated by the moving striations. Although it was impossible to correlate the magnitude of the low-frequency oscillations with the intensity of the sound waves produced by them, a one-to-one frequency correspondence was in evidence under all conditions. Thus, a connection was established between the electrical oscillations in the plasma and acoustic waves in the neutral gas, and a technique was demonstrated for quantitatively investigating the relationship between the two.

This section describes the details of the measurements, presents the significant data, and discusses the results. In addition, possible explanations for the observations appear along with theoretical estimates of the plasma perturbation effect which are derived by using elementary acoustic theory and a simple model for the plasma. Because the low-frequency plasma oscillations, or moving striations, represent such an important influence in the work, a brief summary of their characteristics in dc discharge tubes is also included and used to explain deviations of certain data from what would be expected in a well-behaved plasma.

4.1 Simultaneous Modulation of Plasma and Collision Frequencies

When the sound source is activated in the setup of Fig. 3-1, the plasma-modulated cavity output is crystal detected and fed directly to the wave analyzer. Response curves similar to those shown in Fig. 4-1 indicate that the transducer modulates both the plasma and collision frequencies

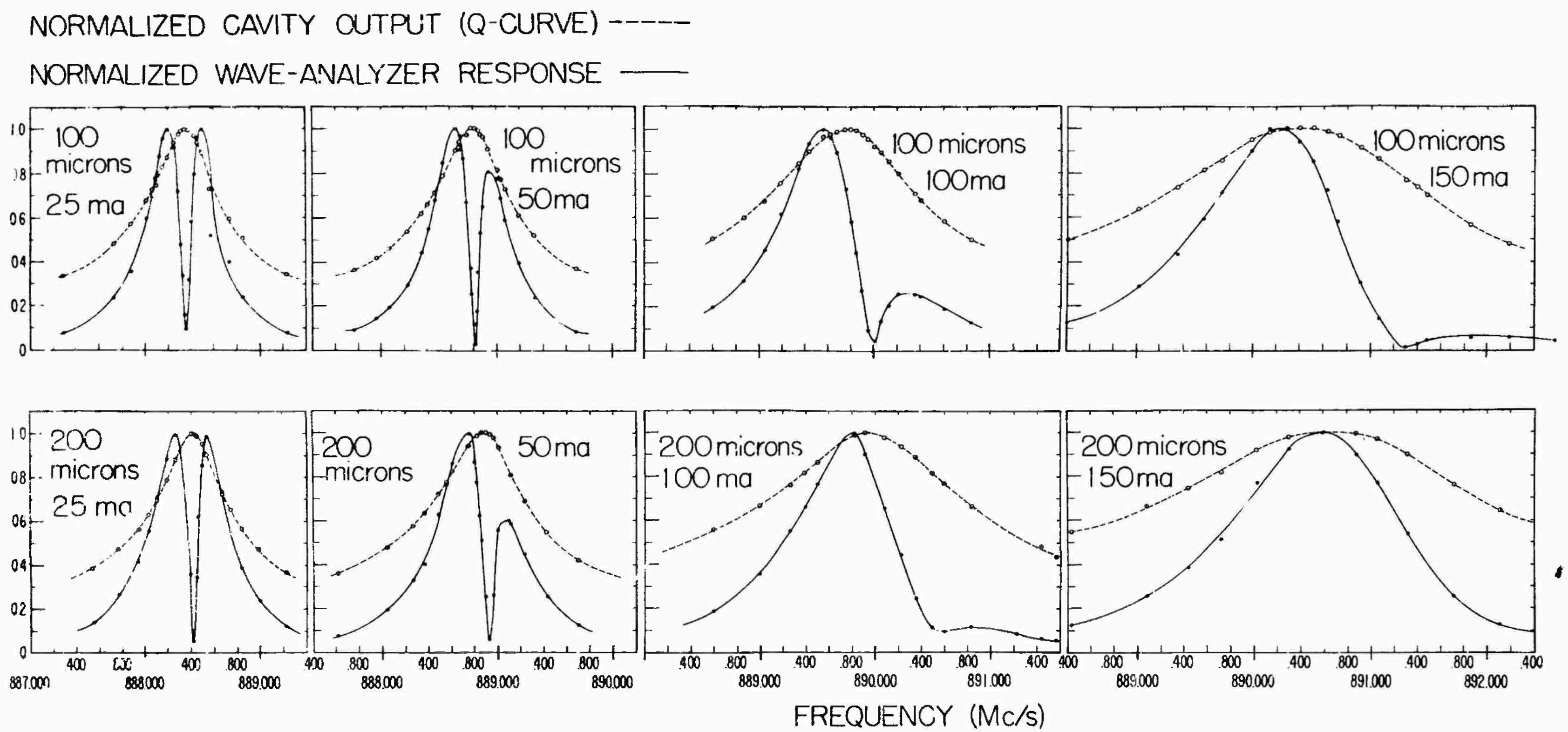

Fig. 4-1. Experimental wave-analyzer responses for acoustically perturbed neon plasmas in dc discharges at typical discharge currents and pressures. In all cases the loudspeaker power is constant ($V_a = 5$ v rms) and the acoustic frequency is 600 c/s. Note the relation of wave-analyzer voltage to the slope of the Q-curve.

simultaneously. This becomes clear by considering that the unperturbed cavity response (Q-curve) for any given discharge [Fig. 4-2(a)] undergoes two separate modulations when the loudspeaker is transmitting. On the one hand, the Q-curve (actually a plot of reciprocal cavity insertion loss vs. frequency) moves back and forth about its equilibrium position due to variations in electron density and, if variations in Q were completely neglected, would produce a wave-analyzer signal whose magnitude and phase at a given electromagnetic frequency would depend on the slope of the Q-curve for all perturbations [Fig. 4-2(b)]. If, on the other hand, the modulation of the Q predominated over the variation in resonance frequency, the wave-analyzer response as a function of the RF would have the same general shape as the Q-curve, with maximum signal at the resonance frequency as illustrated in Fig. 4-2(c). Each point on the Q-curve can be pictured as moving up and down at the acoustic frequency as the pressure variations affect the cavity loss, which, in turn, broadens and then sharpens the Q-curve.

Under the influence of an acoustic perturbation for a fixed RF, the total wave-analyzer voltage $\widetilde{V}$ is equal to the linear superposition of variations in plasma frequency Δf_{pc} and variations in collision frequency $\Delta \nu_c$. To show this, reference is made to Fig. 4-2 and the relations for plasma frequency and collision frequency as a function of cavity-resonance shift and Q [Eqs. (3-32) and (3-33)]. First, under the assumption that $(\nu_c/2\pi f_{or}) << 1$, (3-32) becomes

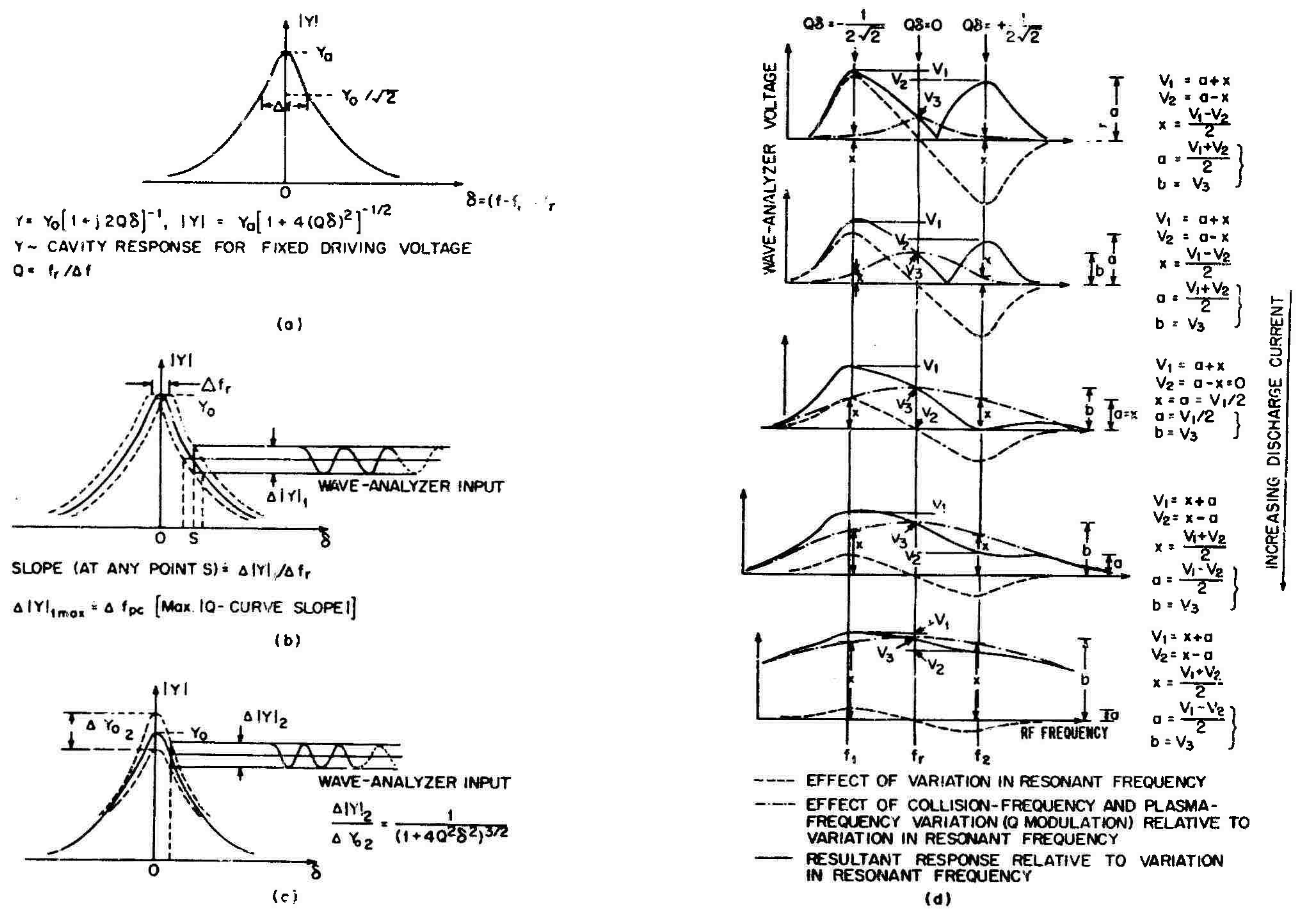

Fig. 4-2. Relations between cavity response and modulation of cavity output due to acoustic perturbations. (a) Typical plot of reciprocal insertion loss in a cavity, and cavity admittance Y as a function of Q and frequency. (b) Effect of resonant-frequency variation only on crystal-detected cavity output. (c) Variation of crystal-detected cavity output due to perturbation of Q. (d) Qualitative explanation of double-humped wave-analyzer response curves showing the effect of Q modulation. By measuring V_1, V_2, and V_3, it is possible to determine the magnitude of variations in plasma frequency (Δf_p) and collision frequency ($\Delta \nu_c$) due to acoustic waves emanating from the loudspeaker. The former is proportional to a, while the latter is related to both a and b.

$$f_{pc} \doteq \left[\frac{f_{or}}{K}(f_r - f_{or})\right]^{1/2} \quad . \qquad (4\text{-}1)$$

Here f_r and f_{or} are the resonant frequencies of the cavity with and without the plasma, respectively; K is a constant. To get the effect of a loudspeaker perturbation for any fixed discharge current (which fixes the f_r), (4-1) is differentiated with respect to f_r so that, after rearranging terms,

$$df_r = \left[2(f_r - f_{or})^{1/2}\sqrt{\frac{K}{f_{or}}}\right] df_{pc} \quad . \qquad (4\text{-}2)$$

For constant discharge current, the bracketed terms are all constant; for small perturbations, $df_r \doteq \Delta f_r$ and $df_{pc} \doteq \Delta f_{pc}$. As a result,

$$\Delta f_r \sim \Delta f_{pc} \, , \qquad (4\text{-}3)$$

which makes the wave-analyzer input $\Delta|Y|_1$ in Fig. 4-2(b) proportional to Δf_{pc} for any specified and fixed RF at which loudspeaker measurements are made. Therefore,

$$\Delta|Y|_1 = A_1 \Delta f_{pc} \, , \qquad (4\text{-}4)$$

where A_1 is a constant.

In Eq. (3-33), both f_r and Q are measured variables under the influence of a transducer perturbation; therefore, the resultant variation in collision frequency is

$$d\nu_c = \frac{\pi f_{or}^2}{(f_r - f_{or})^2 Q}\left[\frac{f_r}{f_{or}}\frac{Q}{Q_o} - 1\right] df_r - \frac{\pi f_{or}^2}{Q(f_r - f_{or})}\frac{dQ}{Q} \quad . \qquad (4\text{-}5)$$

(Q and Q_o are the cavity Q's with and without the plasma, respectively).

Assuming that the loaded cavity can be represented by a series RLC circuit in which the unloaded cavity loss and the loss introduced by the plasma can be lumped into one equivalent resistance, the expression for the Q-curve shown in Fig. 4-2(a) is

$$|Y| = Y_o \left[1 + 4 (Q\delta)^2\right]^{-1/2} \tag{4-6}$$

and may be used to show that

$$\frac{dQ}{Q} = \frac{d\,Y_o}{Y_o} \quad . \tag{4-7}$$

Therefore, for a given discharge current, Eq. (4-5) reduces to

$$d\,Y_o = (\text{const.})_1\, d\nu_c + (\text{const.})_2\, df_r \quad . \tag{4-8}$$

From Fig. 4-2(c), the wave-analyzer voltage $\Delta|Y|_2$ at any given electromagnetic frequency is proportional to ΔY_{o2} ; therefore, if $d\,Y_o \doteq \Delta Y_{o2}$, $d\nu_c \doteq \Delta\nu_c$, and $df_r \sim df_{pc} \doteq \Delta f_{pc}$,

$$\Delta|Y|_2 = B\Delta\nu_c + A_2\,\Delta f_{pc} \tag{4-9}$$

where B and A_2 are constants. Since the resultant wave-analyzer voltage $\widetilde{V}$ is equal to $\Delta|Y|_1 + \Delta|Y|_2$*, then

$$\widetilde{V} = A\Delta f_{pc} + B\Delta\nu_c \,, \tag{4-10}$$

where A is a constant equal to $A_1 + A_2$.

----

*$\Delta|Y|_1$ and $\Delta|Y|_2$ are either in phase or 180° out of phase, depending on the RF at which the measurement is being made. This results from the fact that both voltages are a function of the same pressure variation. Referring to Fig. 4-2(b), when, for example, the pressure increases, the resonant frequency increases and the modulation voltage $\Delta|Y|_1$ increases. For the same pressure increase, the cavity Q lowers and, according to Fig. 4-2(d), $\Delta|Y|_2$ decreases. Similarly, $\Delta|Y|_1$ decreases as the pressure decreases, while the increasing cavity Q causes an increasing $\Delta|Y|_2$ for the situation shown in Fig. 4-2(b) and (c). When the RF frequency is below the resonant frequency, $\Delta|Y|_1$ is in phase with $\Delta|Y|_2$ for all pressure fluctuations.

Descriptions of similar methods for determining small time-varying perturbations in the constitutive parameters of materials have been found in the literature. Kino and Allen [20] used a circular cylindrical cavity resonating in the TM_{010} mode to measure sinusoidal plasma variations resulting from a fluctuating number density in the plasma positive column. Such fluctuations were created by using an audio oscillator connected in series with a steady gas discharge to sinusoidally vary the discharge current. By detecting the modulated cavity output with a crystal, and observing the modulating signal on a cathode-ray oscilloscope, the amplitude of the resultant frequency fluctuation was measured and linearly related to the number density fluctuations; such a procedure corresponds to the measurement shown in Fig. 4-2(b) and the relation described by Eq. (4-4). In addition, Feher, in describing diagnostic techniques for magnetic materials, derived several expressions for changes in the output voltage of a specially designed cavity as a function of both the real and imaginary parts of the RF susceptibility of a sample under test [21]. Feher's objective was to establish conditions which would maximize the cavity-output voltage and signal-to-noise ratio by judiciously choosing the proper RF probing frequencies, sample sizes, and cavity input and output coupling. In the course of his treatment of this problem, he presented an analysis which shows that the change in the output voltage of the cavity is proportional to the change of cavity Q, and that the latter is in turn proportional to either the real or imaginary parts of the RF susceptibility (the real part is proportional to the number density of electrons in the plasma case, while the imaginary part is proportional to the collision frequency). Although Feher's paper did not consider continuous time variations in the susceptibility, such a situation could have been easily introduced and shown to result in an expression for the voltage modulating the cavity RF which was equivalent to Eq. (4-9).

For any discharge current, equilibrium pressure and acoustic intensity, the wave-analyzer response is a superposition of two effects, with their relative importance dependent upon the RF used in the measurement, i. e., upon the constants A_1, A_2, and B. Figure 4-2(d) typifies several ways in which the respective modulations could combine to give the experimental results of Fig. 4-1.

Differentiation of (4-6) with respect to $(Q\delta)$ gives the slope of the cavity response

$$\frac{d|Y|}{d(Q\delta)} = -4Y_o Q\delta\left[1+4(Q\delta)^2\right]^{-3/2}, \qquad (4\text{-}11)$$

which, when differentiated again and set equal to zero, determines the points of maximum slope. If $A_2 = 0$ and $B = 0$, i. e., only modulation of the type shown in Fig. 4-2(b) were considered, maximum loudspeaker effect would be observed at these maximum slopes for electromagnetic frequencies f_1 and f_2 given by $Q\delta = Q(f-f_r)/f_r = \pm 1/2\sqrt{2}$, where f_r and Q are the unperturbed resonance frequency and cavity Q, respectively. Since the wave analyzer measures only the amplitude and not the phase of a signal, a double-humped curve, symmetrical about the cavity resonance, would result from RF sweeping the cavity through its resonance while the loudspeaker was activated. As the effect of the modulation due to variations in

Q becomes important, the symmetry of the wave-analyzer response is lost until, at the other limit when $A_1 \doteq 0$ (low Q's), symmetry is regained, but this time in a response whose form resembles the Q-curve.

In Fig. 4-1, note that the right-hand hump disappears as the current increases at constant pressure due to the fact that the plasma frequency in the discharge tube increases with the current and decreases the Q. When the pressure is reduced, the collision frequency is lowered for any given discharge current so that the influence of collisions on the total response diminishes relative to that of the variations in the plasma frequency. As a consequence, the second hump is enlarged.

By modulating the plasma with small variations in the discharge current instead of with the sound transducer, while keeping all other parameters in the discharge and cavity the same, it was possible to deduce that acoustic waves were indeed exerting an influence on the collision frequency For several typical operating conditions, low-frequency signals from the wave-analyzer BFO were applied to the discharge tube through a transformer in the tube's external driving circuit to simulate the effect of the loudspeaker. In each case, when the cavity was RF swept through its resonance under this perturbation, double-humped responses similar to those in Fig. 4-1 were observed. The magnitude of the current modulation was adjusted such that the peak of the left-hand hump coincided precisely with the corresponding peak which resulted from the double-humped response created by an arbitrarily fixed loudspeaker perturbation.

However, as the RF was swept through the right-hand hump, the latter was seen to be consistently higher than that of the corresponding response due to the transducer. Referring to Eq. (4-5) and Fig. 4-2(d), this observation is consistent with a modulation of the collision frequency by the transducer. Since collision frequency is independent of current in the discharge, the left side of Eq. (4-5) equals zero in the current-modulation case. Thus, the wave-analyzer voltage produced according to Eq. (4-5) [$\sim df_r$] is less than for the transducer perturbation when the presumed effect of $d\nu_c$ adds to the voltage produced by df_r. In the latter situation, then, a greater voltage subtracts from the voltage produced by variations like that in Fig. 4-2(b) than in the case of current perturbation. Figure 4-2(d) helps to show that the result would be a lower right-hand hump for the acoustic modulation. This comparison between current and acoustic disturbances, therefore, places the effect of collision-frequency modulation by sound waves in evidence.

4.2 Pressure Variations in the Discharge Tube

Elementary acoustic theory for an adiabatic process [2] predicts that a piston of area S_p vibrating at frequency f with a maximum deflection θ_o at one end of a closed tube of length ℓ and cross-sectional area S produces a maximum pressure variation Δp_o from equilibrium pressure p_o at any distance $\underline{x}$ from the piston given by

$$\Delta p_o = \frac{\rho_o c^2 \theta_o (S_p/S)}{\sin(2\pi\ell/\lambda)} \frac{2\pi}{\lambda} \cos\frac{2\pi}{\lambda}(x-\ell) , \qquad (4\text{-}12)$$

where ρ_o is the equilibrium density of the medium, c is the speed of sound in the medium, and $\lambda = c/f$. Of course, the instantaneous pressure fluctuates between $p_o + \Delta p_o$ (a compression) and $p_o - \Delta p_o$ (a rarefaction) at the piston frequency. Since

$$c = (p_o \gamma / \rho_o)^{1/2} \tag{4-13}$$

in an ideal gas, where γ is the ratio of specific heat at constant pressure to specific heat at constant volume,

$$\frac{\Delta p_o}{p_o} = \gamma \frac{S_p}{S} \frac{2\pi}{\lambda} \frac{\cos \frac{2\pi}{\lambda}(x-\ell)}{\sin\left(\frac{2\pi}{\lambda}\ell\right)} \theta_o \;. \tag{4-14}$$

If the transducer diaphragm is regarded as a moving piston which transmits sound power into a closed tube with a reflector at one end acting to create a standing wave, (4-14) indicates that pressure variation is proportional to diaphragm deflection for a given acoustic frequency and tube geometry. Using the so-called "frozen composition" assumption of Sodha and Palumbo that the composition of the plasma remains practically unchanged under a periodic acoustic disturbance when the ionization and deionization relaxation times are substantially greater than the period of an acoustic cycle,

$$\frac{\Delta N_e}{N_e} = \frac{\Delta N}{N} \quad \text{(N_e and N are electron and neutral-molecule densities, respectively)} \tag{4-15}$$

is a valid description of the effect of a small pressure variation on the particle density at any point in an ionized gas. Such a description is justified for acoustic frequencies up to about 3 kc/s at a 1 Torr discharge pressure, and up to about 15 kc/s at 0.2 Torr [3, p. 1636]. Since

$$\frac{\Delta p_o}{p_o} = \frac{\Delta N}{N} \tag{4-16}$$

for acoustic waves, it would follow that $\Delta N_e/N_e$ is proportional to θ_o. Therefore, since

$$\frac{\Delta N_{ec}}{N_{ec}} = 2\,\frac{\Delta f_{pc}}{f_{pc}} \qquad \text{(}N_{ec}\text{ and } f_{pc} \text{ are electron density and plasma frequency } \underline{\text{at}}\ \underline{\text{the}}\ \underline{\text{center}} \text{ of the cylindrical plasma column.)} \tag{4-17}$$

from Eq. (1-1), Δf_{pc} should be proportional to the diaphragm deflection for any given discharge condition.

In gas-discharge plasmas of the type used in the experiments conducted in this research, the electron collision frequency ν_c is a function of gas pressure, p_o, average electron velocity, v_e, and the probability of an electron-neutral collision, P_c, according to the relation[17]

$$\nu_c = p_o P_c v_e \,. \qquad \left(\begin{array}{ll} \nu_c & \text{in collisions/sec} \\ p_o & \text{in Torr} \\ v_e & \text{in cm/sec} \\ P_c & \text{in cm}^{-1}\ \text{Torr}^{-1} \end{array}\right) \tag{4-18*}$$

P_c itself is related to the average electron velocity (based on a Maxwellian electron velocity distribution), although conflicting theories and experiments concerning this relationship have not as yet been reconciled. Frequently,

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*This is true if electron-ion and electron-electron collisions are negligible relative to electron-neutral collisions. Assuming an electron temperature of 40,000°K (based on calculation and measurements, this was the electron temperature in the positive column of the discharge used in the experiment), and typical measured electron densities, the calculated electron-ion and electron-electron collision frequencies were in the 1 - 10 kc/s range - very low compared to measured values of ν_c which were usually greater than 100 Mc/s.

in theoretical and experimental plasma work, P_c is assumed to vary with the reciprocal of electron velocity, in which case the collision frequency is independent of the latter. This seems to be a reasonably good approximation in high temperature discharges of certain gases such as hydrogen and helium. However, for this to be the case in the neon used in these experiments, the electron temperature T_e* would have to be greater than approximately 52,000°K. At or beyond this temperature, one could expect the collision frequency to depend only on pressure, so that

$$\frac{\Delta \nu_c}{\nu_c} = \frac{\Delta p_o}{p_o} \quad . \tag{4-19}$$

Over the range of pressures used in the experiments reported here, the collision frequency may not be independent of electron velocity. Referring to theoretical work by Allis and Morse [18] on the collision probability for slow electrons ($V \lesssim 10$ v) in neon, there are three regions in the $P_c - v_e$ relation which have to be considered: $V < 5v$, $5v \leq V \leq 7v$, $V > 7v$. In the former $P_c \doteq K_1 V^3$, where K_1 is a constant. For the latter, $P_c \doteq K_2 \frac{1}{V}$, where K_2 is also a constant (the usual assumption discussed previously). Between 5v and 7v, P_c is just about constant. In each of these cases, small variations in collision frequency are linearly related to small fluctuations in pressure and electron temperature, i.e.,

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* $T_e = \frac{2eV}{3k} = \frac{1}{2} m v_e^2$,

where eV is average energy of electrons in electron-volts, m is the mass of the electron, k is Boltzmann's constant.

$$\frac{\Delta \nu_c}{\nu_c} = \frac{\Delta p_o}{p_o} + K_3 \frac{\Delta T_e}{T_e} , \tag{4-20}$$

where K_3 is a constant which depends on the region of electron temperatures in which measurements are made (for $V < 5v$, $K_3 = 7/2$; for $5v \leq V \leq 7v$, $K_3 = 1/2$; for $V > 7v$, $K_3 = -1/2$).

T_e in the positive column of a glass-enclosed cylindrical dc glow discharge that is controlled by ambipolar diffusion is related to the discharge pressure in the following manner:

$$\frac{\mathcal{E}^{\frac{e}{k}\frac{V_i}{T_e}}}{\sqrt{eV_i/kT_e}} = 1.16 \times 10^7 C^2 p_o{}^2 R^2 , \tag{4-21}$$

where e is the electronic charge, V_i is the ionization potential of the discharge gas in volts ($V_i = 21.5$ v for neon), p_o is discharge pressure in Torr, R is the radius of the discharge in centimeters, k is Boltzmann's constant, and C is a constant equal to 5.9×10^{-3} for neon [19]. This relation was used to show that the region for which $P_c = K_1 V^3$ corresponds to gas pressures greater than 0.30 Torr.* For the collision frequency to be independent of electron velocity, Eq. (4-21) indicates that the pressure has to be less than about 0.14 Torr. For intermediate pressures, $P_c \doteq$ constant and $\nu_c \doteq (\text{const.}) p_o v_e$. In all cases, however, Eq. (4-21) shows that

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*In this region $T_e \leq 38,600^\circ K$.

$$\frac{\Delta T_e}{T_e} = (\text{const.}) \frac{\Delta p_o}{p_o} \tag{4-22}$$

for any given discharge under the influence of small pressure perturbations produced by sources such as acoustic transducers. From Eq. (4-20), therefore,

$$\frac{\Delta \nu_c}{\nu_c} = (\text{const.}) \frac{\Delta p_o}{p_o} \quad , \tag{4-23}$$

which indicates that $\Delta \nu_c$ should be proportional to the diaphragm deflection for a given discharge.

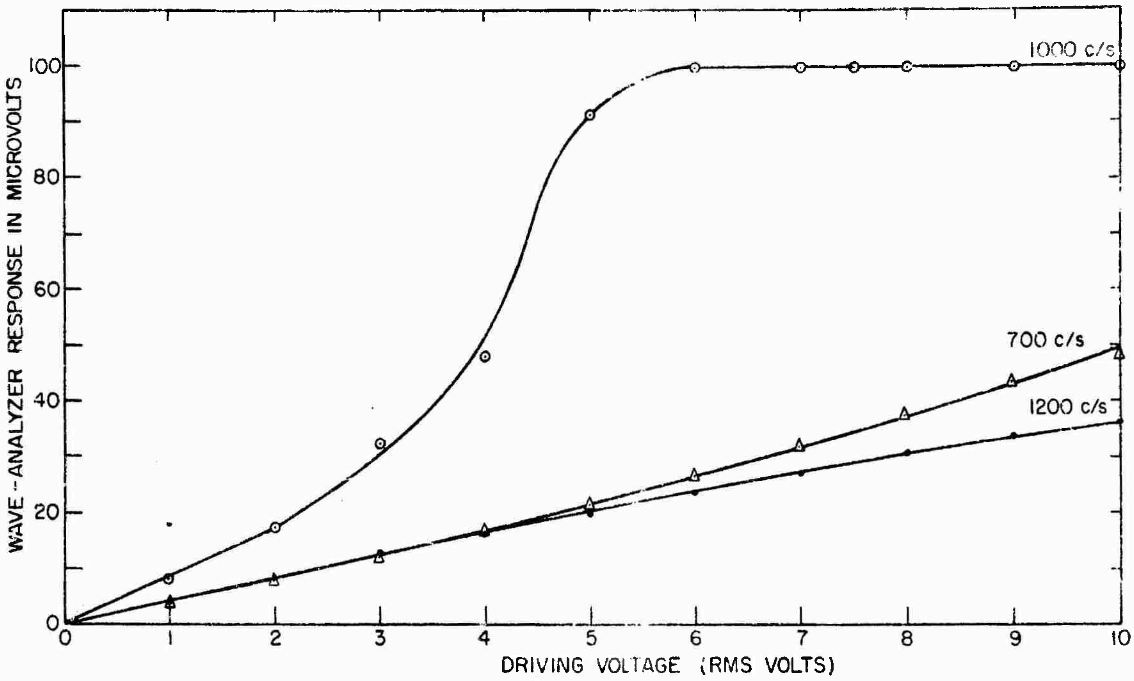
The line of reasoning just presented relies strongly on the dependence of P_c on v_e. It must be emphasized that there are often wide discrepancies between measured values of P_c and theoretical values, even in the best correlations between the two. If one is unfortunate enough to be working in a range of electron temperatures which is unsuitable for predicting $P_c(v_e)$, the above analysis might not be so straightforward. Since the discharge pressure is an influence in determining this range, great care must be taken to choose a value for pressure which will minimize the vagaries in collision probability. At pressures less than 0.15 Torr, P_c is probably quite predictable; for pressures greater than 0.30 Torr, there is apt to be difficulty in estimating the behavior of the electron-neutral collision frequency. For the measurements in Fig. 4-3 made at 0.2 Torr, $P_c \doteq$ constant.

Bearing in mind that the detection system is sensitive to the sum of linear terms in Δf_{pc} and $\Delta \nu_c$, the resultant wave-analyzer responses should be proportional to the diaphragm deflection if the simple acoustic theory and assumptions which were developed earlier in this section are adequate. Wave-analyzer responses in Fig. 4-3 do, indeed, show a close point-by-point correspondence to the diaphragm-deflection curves in Fig. 2-5 as a function of loudspeaker driving voltage to substantiate this prediction. Furthermore, the correlation between Figs. 4-3 and 2-5 is consistent with the descriptions of relative change in plasma and collision frequencies given in Eqs. (4-15) and (4-23).

4.3 Magnitude of Plasma-Frequency Variations

Determining the magnitudes of Δf_p and $\Delta \nu_c$ for any given discharge and transducer operation involves the separation of the two effects in the wave-analyzer signals. A calibration scheme has been developed for this purpose which takes advantage of the double-humped responses discussed in 4.1. Although Δf_p and $\Delta \nu_c$ can both be found using similar procedures, the initial interest was in the former, and a brief explanation of how it was determined follows.

First, the cavity-perturbation method described in 3.2.2 was used to make a static measurement of plasma frequency at the discharge-tube axis (f_{pc}) for two discharge currents which were slightly above and below the



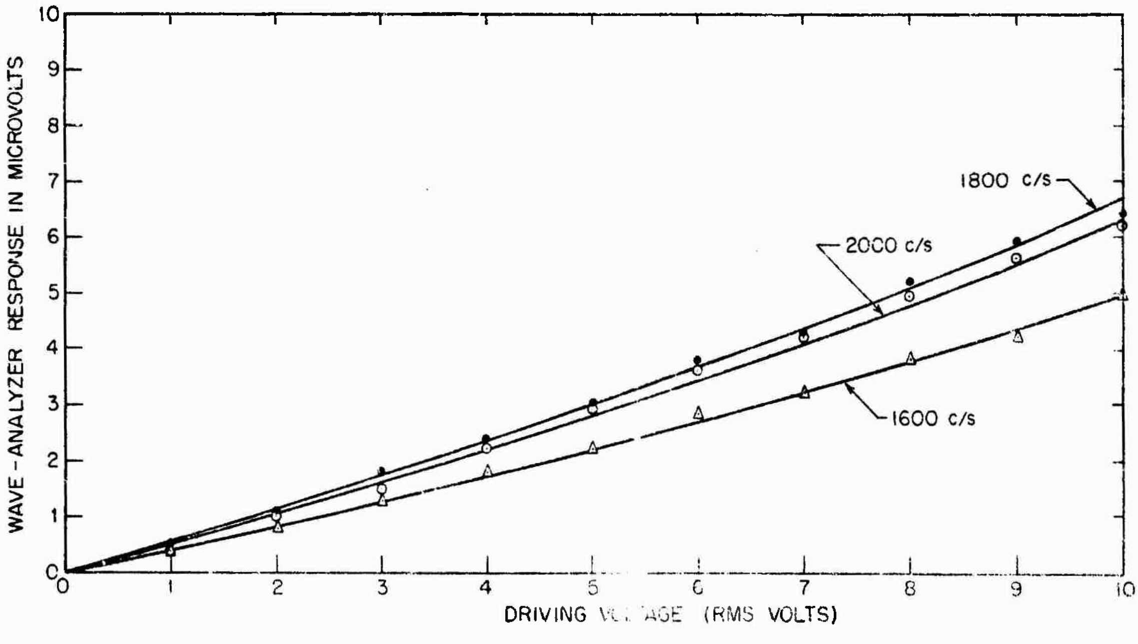


Fig. 4-3. Response of a 100-ma neon discharge at 0.2 Torr to changes in acoustic intensity caused by variations in loudspeaker driving voltage V_a. RF is constant equal to 889.963 Mc/s. These data correlate with those of Fig. 2-5 to show that wave-analyzer response is proportional to diaphragm deflection.

original current; then $(f_{pc_2} - f_{pc_1})/(I_2 - I_1) = \Delta f_{pc}/\Delta I = k_p$ was computed.* (Actually, more accurate values of k_p can be found for any discharge current by differentiating the empirical relationship between f_{pc} and the discharge current as determined by independent measurements such as those in Fig. 3-6.*) A low-frequency signal from the wave-analyzer BFO was applied to the discharge tube through a transformer in the external driving circuit, and the magnitude of the variation in the discharge current measured by feeding the signal appearing across the external discharge resistor to the analyzer input (see Fig. 3-1). This external signal modulated the entire discharge uniformly and simulated the modulation of the plasma frequency by the loudspeaker in that portion of the positive column probed by the cavity.

Figure 4-4 indicates that the BFO frequency used in this calibration is critical and, generally, must be below approximately 200 c/s. Curves in Fig. 4-4 were produced by adjusting the BFO output so that the voltage across the 500 Ω load resistor was constant (i.e., the current modulation in the discharge tube was held constant), and measuring the crystal-detected cavity output on the wave-analyzer for various average discharge currents. The dip and apparent resonance in each curve as the BFO frequency is varied show that some process is involved in this type of plasma modulation other than a uniform fluctuation of electron density in the positive column of the

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*In the computations that follow, possible corrections to the f_{pc} data (discussed in Section 3.2) have not been included.

discharge. Such an effect can most likely be attributed to a so-called glow-discharge resonance reported by Stewart [3] and others [4, 5] which is closely linked to the excitation of moving striations in dc discharge tubes modulated by audio-frequency ac signals. These investigators observed visible striations and standing light patterns when the frequency of the alternating potential was of the same magnitude as the freque y the spontaneous oscillations which were present in the gas when it s excited with an ordinary dc potential ($\approx$ 200 c/s to $\approx$20kc/s). Since such striations correspond closely to marked variations in electron density, it is quite likely that frequency-sensitive inhomogeneities in the region of the positive column sensed by the cavity were responsible for the observed variation of cavity response in Fig. 4-4. In any event, because k_p was computed on the basis of a static measurement, it seemed reasonable to approach this condition as closely as possible in the calibration. Therefore, BFO frequencies below 200 c/s were used where (as can be seen from Fig. 4-4) the cavity output was independent of frequency.

With reference to the calibration procedure, $\underline{a}$ in Fig. 4-2(d) is defined equal to $\Delta|Y|_{1max}$ in Fig. 4-2(b), which makes Δf_{pc} proportional to $\underline{a}$ in an even better approximation than the general proportionality that exists between $\Delta|Y|_1$ and Δf_{pc} for <u>any</u> Q-curve slope(at any RF). Since $\underline{a}$ is a maximum at the two frequencies f_1 and f_2 determined by $Q\delta = \pm\ 1/2\sqrt{2}$, the RF was adjusted to these values after a separate

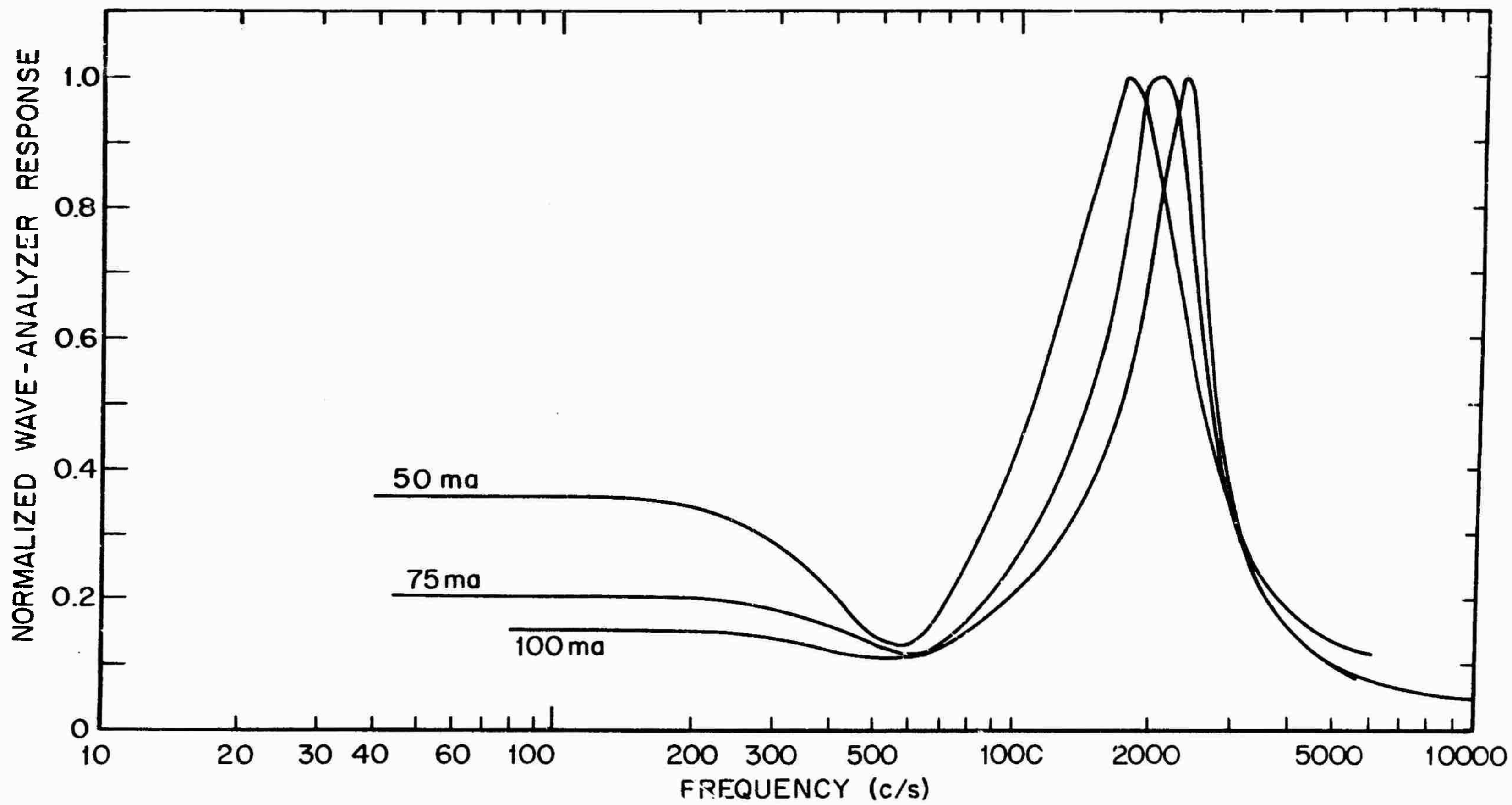

Fig. 4-4. Apparent resonance in a continuous dc neon discharge which is modulated by an audio signal. Alternating current variation in the discharge is fixed at a value corresponding to 2 mv across a 50C ·d resistor. Discharge pressure = 0.2 Torr; cavity RF is near the resonance of ₁09 mode (approximately 889 Mc/s). Each curve represents normalized crysta ·ed cavity output for typical average discharge currents.

determination of Q and resonance frequency, and wave-analyzer readings were taken. Note from Fig. 4-2(d) that $a = (V_1 + V_2)/2$ [or $(V_1 - V_2)/2$ depending on relative effects of the electron density and Q variations].

Next, the BFO was disconnected and the loudspeaker turned on to a given sound power and frequency. The RF was again set at f_1 and f_2, the wave-analyzer tuned to the acoustic frequency involved, and the corresponding responses V_1 and V_2 measured (now called V_1' and V_2', respectively). Since it is known that Δf_{pc} is proportional to $V_1 + V_2$ (or $V_1 - V_2$), and that a known current variation $I_2 - I_1 = \Delta I$ from the wave-analyzer BFO produces a known Δf_{pc} determined from a knowledge of k_p, it follows that

$$\Delta f_{pc}(\text{caused by the loudspeaker}) = k_p \Delta I \frac{V_1' + V_2'}{V_1 + V_2} \left(\text{or} = k_p \Delta I \frac{V_1' - V_2'}{V_1 - V_2}\right) . \tag{4-24}$$

With this calibration procedure, Δf_{pc} was measured as a function of plasma frequency for constant diaphragm deflection at several acoustic frequencies. Equations (4-14), (4-16), and (4-17) predict that Δf_{pc} should be proportional to f_{pc} for any acoustic frequency and sound intensity, provided that $\Delta N_e/N_e$ is equal to $\Delta N/N$. Figure 4-5 bears this out for low plasma frequencies, although each curve shows a pronounced departure from a linear relationship at higher plasma frequencies, with "dips" becoming more pronounced with increases in acoustic frequency. If consideration is given only to the linear region for a moment, a comparison of $\Delta f_{pc}/f_{pc}$ determined experimentally with that predicted by simple acoustic theory shows fairly good agreement, although such correlations are not necessarily conclusive because of possible plasma side effects and departures of the discharge tube construction from that assumed in the simple acoustic model.

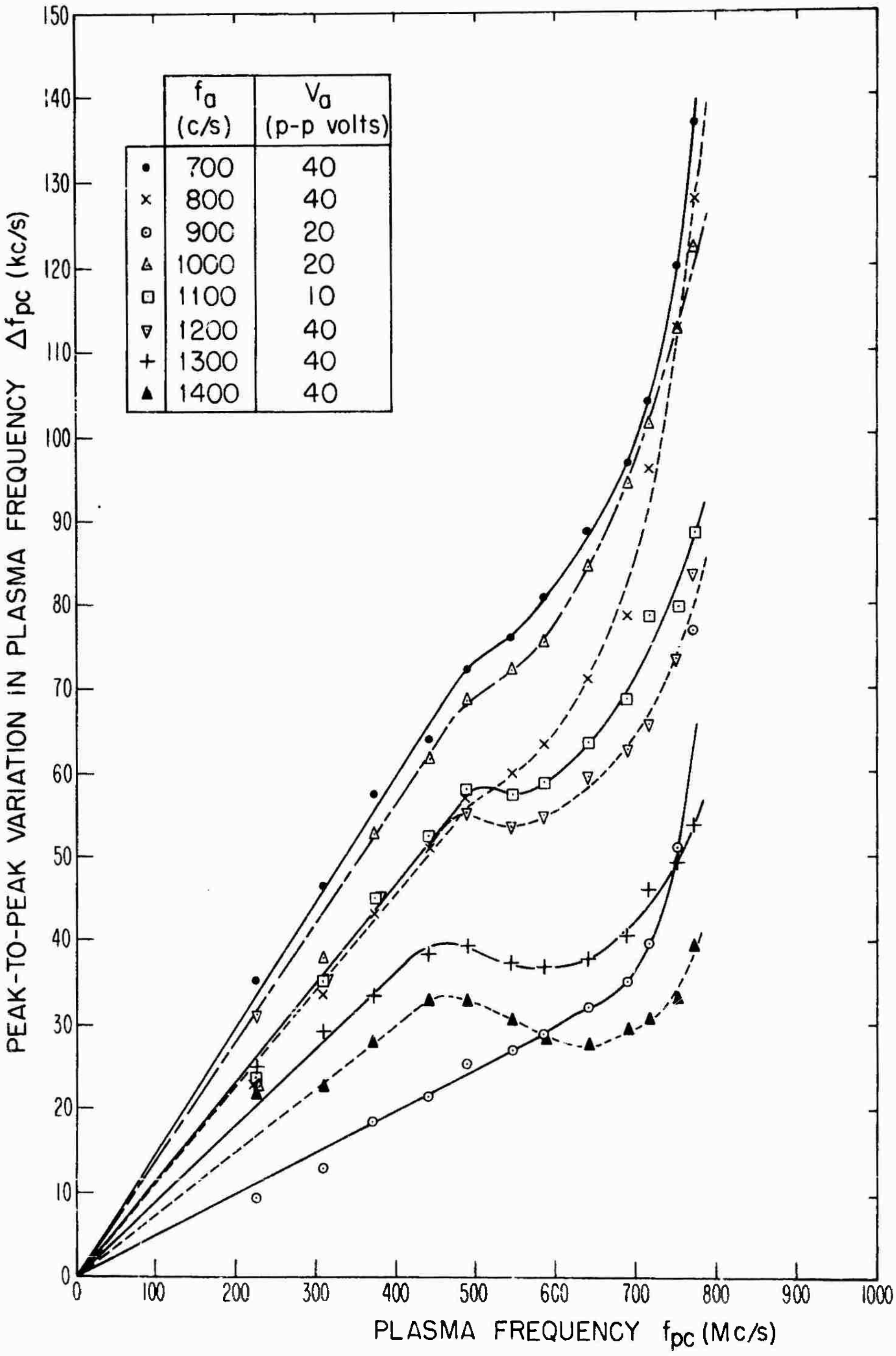

Fig. 4-5. Effect of acoustic waves on the plasma frequency of a neon discharge at 0.2 Torr. Loudspeaker driving voltages V_a are constant at the values indicated for each acoustic frequency f_a, and correspond to diaphragm deflections determined from Fig. 2-5. Cavity operation is in the TE_{109} mode near 888 Mc/s.

For example, slopes of the 700 and 1000 c/s curves in Fig. 4-5 are 1.5×10^{-4} and 1.4×10^{-4}, respectively, corresponding to 2.1×10^{-4} and 1.0×10^{-4} as calculated from Eq. (4-14) with diaphragm deflections taken from Fig. 2-5. In Eq. (4-14), the equivalent piston area of the diaphragm, vibrating with maximum amplitude θ_o at its center, is taken as 42 % of the area inside the diaphragm seating surface [7] making the ratio $S_p/S = 0.1376$. The equivalent length of the discharge tube from the vibrating diaphragm to the sound-reflecting anode is approximately 56 cm, and the cavity probes in the tube in the positive column over a volume whose average distance from the anode is 12 cm. Also, the ratio of specific heats in neon is 1.64 and the calculated sound velocity is about 440 m/sec corresponding to measured temperatures of 22-26°C on the outside wall of the discharge tube at the position of the positive column.

At or near the calculated acoustic resonances of the ideal closed cylindrical tube, the correlation between theory and experiment is poor — deviating by as much as an order of magnitude. This is most likely due to the departure of the theoretical resonances from those that actually exist in the tube.

Data plotted in Fig. 4-6 under the same conditions as in Fig. 4-5, except at a lower discharge pressure, reveal a pattern similar to the 0.2 Torr measurements. Equation (4-14) shows that the relative change of pressure $\Delta p_o/p_o$ is independent of the ambient pressure, which suggests that $\Delta f_{pc}/f_{pc}$ in the linear regime at 0.1 Torr in Fig. 4-6 should agree with that at any pressure. Figure 4-5 provides a basis for comparison of $\Delta f_{pc}/f_{pc}$ measurements at another pressure (0.2 Torr), although for only a few acoustic frequencies. However, the summary of these measurements which appears in Table 4-1, along with calculated values, indicates sufficient correlation to substantiate the theoretical model of what takes place in the plasma.

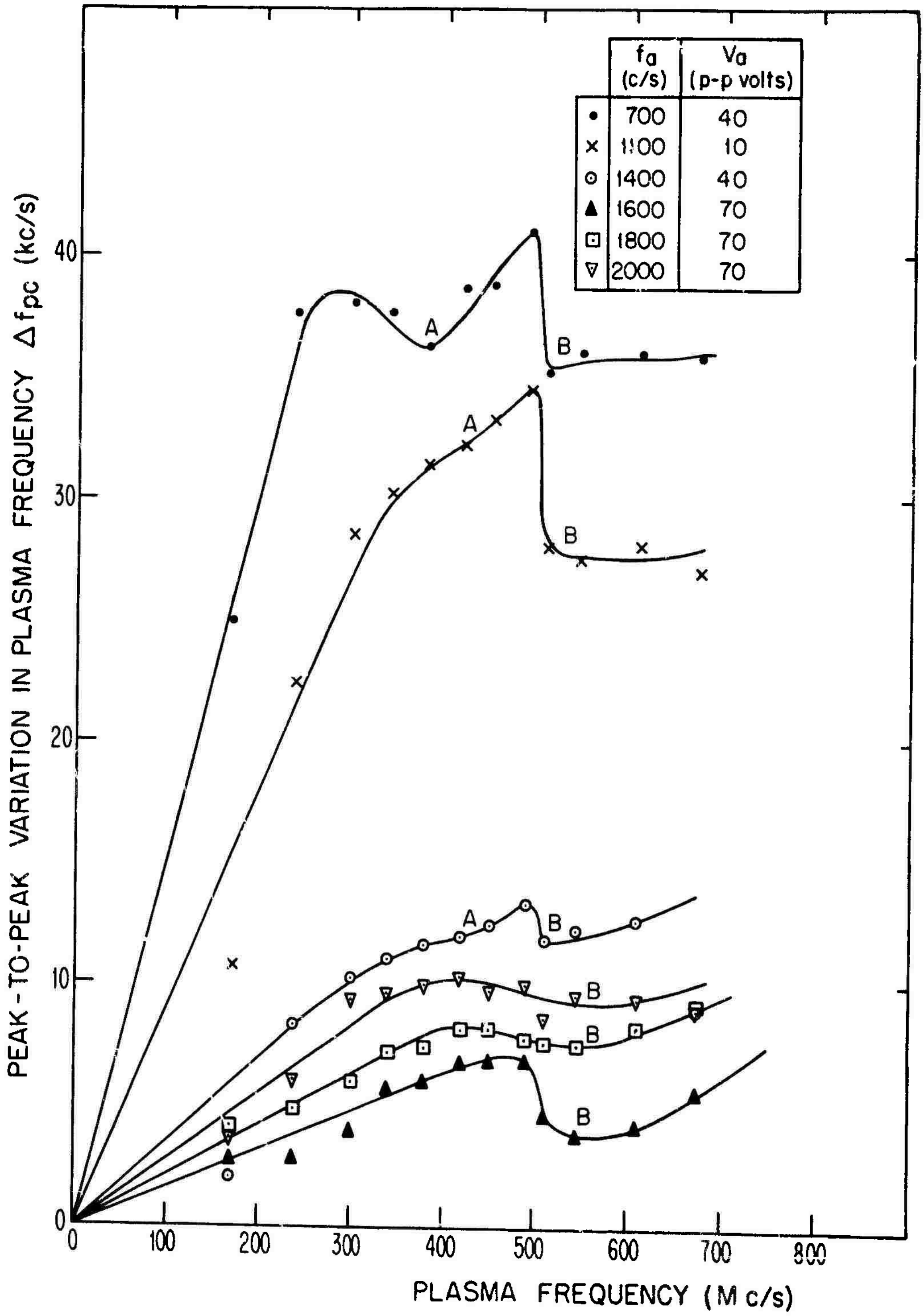

Fig. 4-6. Effect of acoustic waves on the plasma frequency of a neon discharge at 0.1 Torr as a function of loudspeaker frequency. Other experimental conditions are the same as in Fig. 4-5.

TABLE 4-1

Relative Change of Plasma Frequency ($\Delta f_{pc}/f_{pc}$)

	ACOUSTIC FREQUENCY		
	700 c/s	1100 c/s	1400 c/s
0.1 Torr	1.6×10^{-4}	0.9×10^{-4}	0.4×10^{-4}
0.2 Torr	1.5×10^{-4}	1.1×10^{-4}	0.7×10^{-4}
Calculated	2.1×10^{-4}	0.9×10^{-4}	0.8×10^{-4}

Figure 4-6 also exhibits linear relationships between Δf_{pc} and f_{pc} for low plasma frequencies and dips at the higher plasma frequencies, but this time there is an additional dip at intermediate plasma frequencies which is most pronounced in the 700 c/s data. Possible explanations for this odd behavior in both Figs. 4-5 and 4-6 include acoustic coupling with natural low-frequency plasma oscillations, degradation or enhancement of RF probing signals due to energy transfer into modulation sidebands of the original frequency [8], coupling of plasma column RF resonances to the cavity, acoustic resonance of the glow discharge itself when driven at or near critical frequencies, the effect of the medium's ambient temperature variations on sound velocity in the neutral gas (and subsequently on the acoustic standing-waves), and the departure of the sound velocity from the predicted value due to the effect of viscous friction in the acoustic wave propagation [9]. Of these effects, preliminary measurements and analyses indicate that the interaction between excited moving striations and the propagating sound waves from the transducer is most likely responsible for the dips. Furthermore, if one looks closely at Fig. 4-5, there are also enhancements in the $\Delta f_{pc}-f_{pc}$ curves at the highest plasma frequencies; these, too, could be attributed to a coupling mechanism. To achieve a better feeling for the possible interaction, a brief description of spontaneous low-frequency plasma oscillations will now be given along with a summary of their similarities to artificially induced moving striations.

4.4 Traveling Density Variations in Discharge Tubes

The literature abounds with reports related to the investigation of disturbances in glow-discharge tubes which are accompanied by periodic, moving variations of the light intensity in the positive column. Although such unsteady behavior is usually undetectable by the human eye because the velocity of the motion is very high, the fluctuations are characterized by oscillations in the terminal voltage and current of the discharge which are relatively smooth, periodic functions of time. The oscillations correlate with disturbances within the discharge tube which are also periodic in both space and time. However, a complete theory for this phenomenon is still lacking, although the recent work by Cooper at M. I. T. [1] on the experimental as well as theoretical aspects of this problem seems promising in explaining the mechanism of the moving striations.

Several investigators have found that periodic variations in the electron density, electron temperature, longitudinal electric field intensity, and terminal voltage and current of the tube, accompany the periodic variations in light emanating from the positive colum ` [10]. Since the best observations of the moving striations have been with a phototube which senses the light emitted by excited atoms in the discharge, it is quite probable that a wave of ionization accompanies the light. Furthermore, from the variation in the electric field and the electron density, a variation in the ion density with space and time has also been postulated. Thus, it seems that all the measurable properties of the plasma are disturbed by the presence of the wave. Most theories developed to date which account for the existence of moving striations in glow discharges have been linearized theories, even though almost all of the experimental evidence points to the conclusion that naturally occurring moving striations are a large-signal phenomenon in the plasma. In exhaustive experimental studies, Cooper, using Langmuir probes and photodetectors, reaffirmed the large-signal nature of the oscillations and observed that the moving density waves travel toward the anode with velocities on the order of 5×10^5 cm/sec. Similar studies by others indicate that moving striations can also move toward the cathode, and in fact, positive and negative striations may exist in a discharge simultaneously, although the motion of such waves toward the cathode is invariably at a much lower velocity than those which move toward the anode.

One of the most significant results of Cooper's work was the establishment of stable, unstable, and quasi-stable regions of discharge-tube operation. For certain regions in the pressure-current plane of a dc discharge, moving striations are absent and, in this so-called stable condition, the conventional static theory of a positive column is appropriate. However, there are also unstable regions in which spontaneous oscillations occur, and here the static theory must be modified. When the geometry of the tube is set and the type of gas to fill the tube selected, these regions are fixed at positions which are themselves quite stable. At values of current and pressure which place the operation of a glow-discharge tube in a stable region, but near the boundary of an unstable region, the plasma of the positive column is in a quasi-stable state. Minute noise fluctuations, which are always present in the plasma, become larger in amplitude, and small but measurable random fluctuations appear in the terminal voltage and current. Cooper found that an impulse-current perturbation applied to the discharge tube in a pressure - current region near spontaneous oscillation caused a propagating disturbance within the plasma. This disturbance had a striking resemblance to natural moving striations. He, therefore, carried out exhaustive experiments in the quasi-stable regime, where the nonlinear characteristics of the large-signal phenomenon were absent from the induced disturbance. Upon application of the current impulse to the discharge tube, a wave packet was observed (manifested by a coherent oscillation in the light intensity above the average) which traveled toward the anode with a velocity between 10^4 and 10^5 cm/sec which he called "group velocity." Within this packet was a sub-structure which traveled in a direction opposite to the packet itself toward the cathode with a velocity termed the "phase velocity." Typically, the packet itself spreads out as it travels axially, and its amplitude may decay with distance, remain relatively constant, or grow as it makes its way toward the anode. Which of these conditions occurs is determined by the proximity of the mode of operation to the unstable region. The closer the unstable region of operation, the less attenuation; when very near the region of instability, operation may occasionally pass over to uncontrolled spontaneous oscillation. In addition to the opposite direction of the group and phase velocities, the group velocity is always greater than the phase velocity by a factor that lies between 2 and 10, according to Cooper's measurements. Typical wavelengths associated with the structure are of the order of centimeters, which compares closely to that of natural positive striations - those moving toward the cathode. This is the same relationship as that between the magnitude of the velocities of the natural positive and negative striations in a discharge. Using observations in a quasi-stable regime as a basis, Cooper also developed a linearized one-dimensional theory which led to the result that an ionization-induced instability could be predicted in glow-discharge plasmas under conditions that are similar to those found in experiments in which the plasma is unstable. Although his theory is still somewhat incomplete, he was able to form the conclusion that the observed traveling density waves constitute a phenomenon related to ionization and, hence, termed them ionization waves.

Previous work on moving striations strongly suggests an interaction between externally induced sound waves and those emanating from the natural oscillations. There was no guarantee that the data of Figs. 4-5 and 4-6 were taken when the discharge was in a truly quiescent condition. Although a sampling of the discharge-tube current on the most sensitive scale of an oscilloscope indicated the absence of spontaneous oscillations, they could have, nevertheless, been present as very low-level signals, and may have interacted with the sound waves propagated into the positive column. There were, in fact, indications from time to time that, for certain currents and pressures, the discharge was just on the verge of a large-signal spontaneous oscillation, but would remain fairly stable at a position in which an oscillation was just barely observable on the oscilloscope. If such an oscillation were present, even weak coupling of charged particles to the neutral gas in the slightly-ionized medium could foster the citation of a sound wave which, according to theory developed by Cooper, would be dependent on the time rate of the ion density variation. Since the externally forced sound waves presumably also produced variations in ion density, an interaction was a distinct possibility.

Another factor contributing to an interaction of the two acoustic waves is a consequence of the unusual placement of the cathode in the T-section of the discharge tube. Usually, instabilities in the discharge occur within a single continuous range of currents at each pressure, but when the electrodes are irregular, or there are sharp bends in the path of the discharge, moving striations may be observed in numerous and complex regions in the pressure-current plane [11]. Therefore, in examining the geometrical configuration of the experiments reported here, one would expect to find that the medium in thevicinity of the cathode (where the discharge "bends" around the T-extension into the main body of the tube)is quite sensitive to disturbances - either electrical or mechanical. Although the stimulation of sound waves in the neutral gas by a moving ion perturbation is probably not great, coupling in the reverse direction could be considerable, and possibly could result in the excitation of traveling variations in density similar to those excited by Cooper using electrical impulses. This wave motion, and the waves emanating from the transducer, would have a significant time to interact before they reach that portion of the positive column probed by the cavity and, therefore, their combined effect would be observed, especially if their respective frequencies, phase velocities, and amplitudes were appropriately aligned. In Fig. 4-5, the interaction — as measured by the departure from linearity — occurs for the higher acoustic frequencies and seems to be quite pronounced around 1400 c/s, although the range of data presented is not great enough to permit the conclusion that this is the frequency of maximum coupling. Since the increase in plasma frequency is caused by an increase in discharge-tube current, the latter may be approaching a region of instability in the current-pressure plane, and may account for the departure from linearity as the plasma frequency increases. Additional evidence for an interaction mechanism is inherent in the fact that the data indicate an acoustic coupling at a frequency for which spontaneous oscillations often occur.

Such observations are consistent with measurements by Cooper [12] which show that the frequency of natural and induced traveling variations in ion density increases with discharge-tube current (for a fixed pressure). This would suggest an increase in the strength of the interaction at higher plasma frequencies and, based on the data of Fig. 4-5, gives additional support to the proposed interaction. Figure 4-6 presents further data which confirm this explanation. Here, there is strong evidence of coupling in the vicinity of 1600 c/s. Referring to the regions marked with a B in each curve, the dip is seen to increase with acoustic frequency up to 1600 c/s and then to begin leveling out in the 1800 c/s and 2000 c/s data.* In addition, there is strong evidence for an interaction somewhere in the vicinity of 700 c/s as indicated by the regions marked A in Fig. 4-6. This coupling just begins to become visible in the 1400 c/s curve, and is most obvious in the 700 c/s data. Unfortunately, data for lower acoustic frequencies were not taken, so that the point of maximum coupling is not very well defined. However, the trend seems clear.

As a final observation on the proposed coupling, the data in Fig. 4-5 for plasma frequencies beyond 700 Mc/s suggest the possibility of an actual <u>enhancement</u> of the sound wave propagated into the positive column of the plasma. Note the rapidly increasing slopes of the curves in this region, indicating large departures from the Δf_{pc} values which would be predicted

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*Data in Fig. 4-6 are not normalized, so that the <u>depth</u> of each dip is relative only to the remainder of the associated curve.

by extrapolating the linear portion of each curve (for which, presumably, no interaction occurs) into higher plasma frequencies. This "amplification" would certainly be in line with the proposed interaction based on the fact that the suspected natural oscillations could be oriented, in both space and time, in such a way as to result in addition of their effect with that of the forced sound wave.

Figure 4-7 gives some idea of the havoc that the moving striations can create during attempts to observe sound-wave perturbation in the discharge tube. By carefully adjusting the pressure, it was possible to perform experiments very close to the boundary between stability and instability in the discharge tube, i.e., firmly in the quasi-stable region. Propagation of sound waves into the discharge under this condition produced multiple effects similar to those in Figs. 4-5 and 4-6. The same reasoning as before suggests that there are several acoustic-frequency interactions beyond the reasonably linear region at low plasma frequencies. This time, however, some of the curves have small dips in addition to the large interactions of the type observed previously. In attempting to explain Fig. 4-7 in terms of possible acoustic coupling, it seems reasonable to speculate on the existence of large interactions in the regions marked A, B, and possibly C. Coupling denoted by A's first makes itself evident in the 700 c/s data and increases in strength up to about 1700 c/s, whereupon it seems to diminish (relatively) in the 2000 c/s data. Note that the dip in each curve for progressively higher acoustic frequencies is situated at correspondingly higher plasma frequencies as in Figs. 4-5 and 4-6. A similar interaction

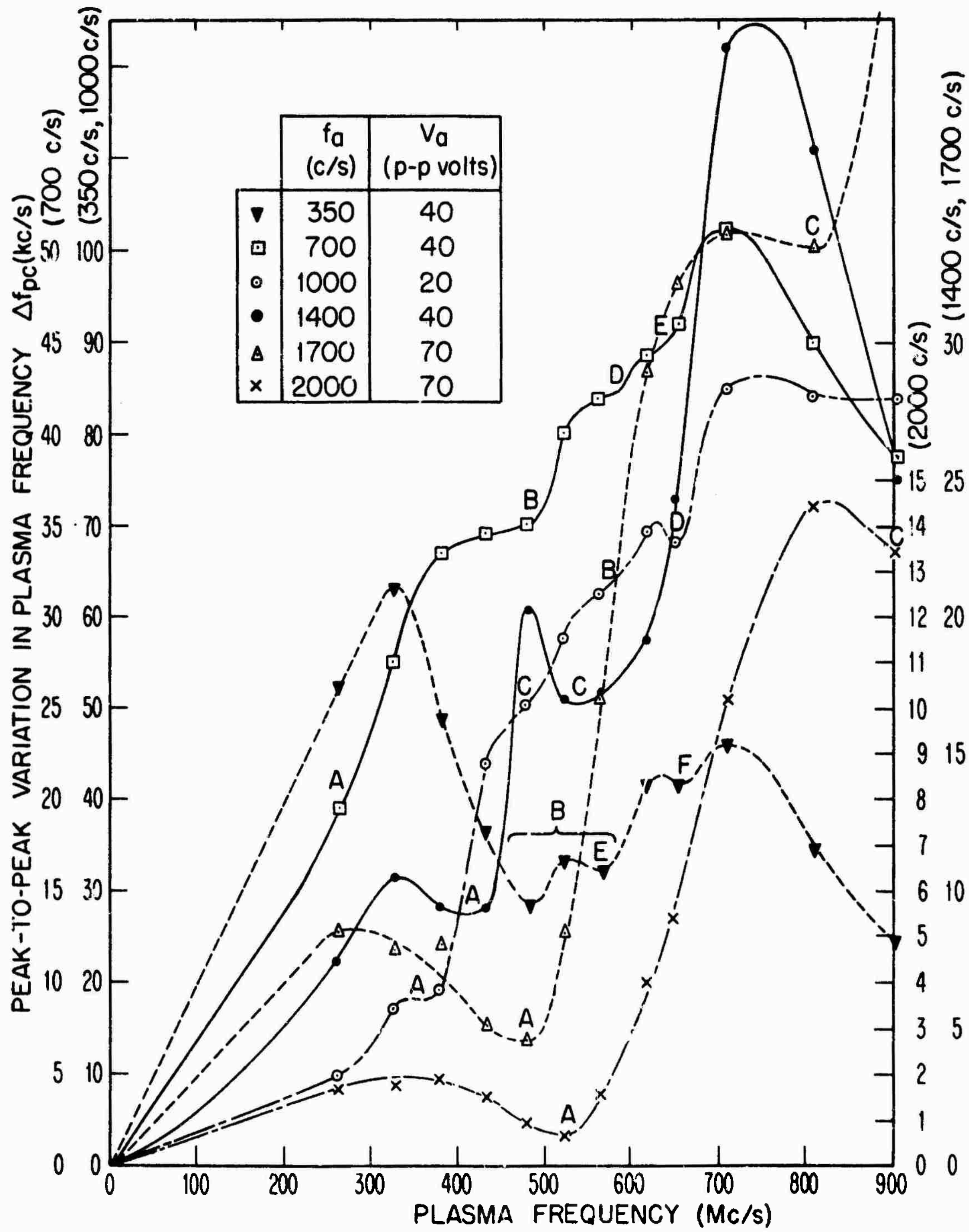

Fig. 4-7. Effect of acoustic waves on the plasma frequency of a neon discharge operating in a suspected quasi-stable region of the current-pressure plane. Ambient pressure = 0.25 Torr; cavity operation is in the $TE_{10\,11}$ mode near 1026 Mc/s.

(regions denoted by B) is seen to occur at higher plasma frequencies, beginning in the 350 c/s curve, but apparently disappearing beyond 1000 c/s. The large dip C in the 1400 c/s data may have had its start in the vicinity of 1000 c/s, conceivably shows up at the point indicated in the 1700 c/s curve, and perhaps manifests itself again in the 2000 c/s data in a region which appears to be developing beyond the range of plasma frequencies for which data were taken. However, all of the curves show that some effect is taking place beyond the observed plasma frequencies, making it impossible to establish any concluding trend for the presumed interaction.

As for the small wiggles, they seem to be superimposed on the larger dips, and tend to behave in the same manner. Dip D in the 1000 c/s data, for example, may have actually manifested itself in the 700 c/s curve (at a lower plasma frequency, as one would expect), but have substantially died away by the time that 1400 c/s was reached. Regions marked E and F in the 350 c/s curve seem to have almost disappeared in the 700 c/s data, although the former could have shown up at a higher plasma frequency in the area indicated. In general, the smaller dips do not seem to manifest themselves over as broad an acoustic range as the larger interactions, and are much less marked.

As the plasma frequency increases, so does the possibility that the plasma oscillations will vary in frequency and phase velocity. Furthermore, since the natural oscillations are usually quite rich in harmonics, interactions between the forced sound waves and harmonics of the plasma oscillations are also a possibility. In any event, measurements, of which Fig. 4-7 is typical, lead to the conclusion that the departure from an ideally linear relationship between Δf_{pc} and f_{pc} increases near the boundary between stability and instability of the discharge.

4.5 Oscillation-Induced Sound Waves

Although the presence of natural low-frequency oscillations in the discharge somewhat hindered attempts to determine, experimentally, the effect of externally induced sound waves in the plasma, the setup for the latter study lent itself to at least a superficial investigation of the acoustic properties of the former. Exact descriptions of the origin and nature of the moving striations are still not available, but investigators looking into this phenomenon frequently hint that the fluctuations either resemble sound waves or induce sound waves through collisions between charged and neutral particles. Cooper, in his early work on moving striations [13], tried to observe such acoustic waves with an ordinary crystal microphone placed externally against a discharge tube, but could not detect them, either because they were very weak and could not propagate through the glass wall, or because there was no radial propagation of sound waves. However, people working with dc discharges frequently hear sound emanating from them, although, to the author's knowledge, none has ever made quantitative measurements of this effect. For this reason, a transducer was used as an uncalibrated microphone to establish the presence of sound waves under conditions when low-frequency oscillations did occur. A direct correspondence between the two was established.

When using the transducer passively as a microphone, the field coils are connected directly to a 40-dB preamplifier whose output is measured with a Hewlett-Packard 302A wave analyzer. The wave analyzer is tuned manually through the frequency range of interest, and the magnitude of the discrete responses are recorded. After this measurement, the frequency spectrum of the current fluctuations in the external circuit of the discharge tube is similarly observed. Frequency spectra of typical current fluctuations observed in discharges appear in Fig. 4-8 with simultaneous observations of microphone output. Also shown in each case is the waveform of the oscillation as observed in the external circuit of the discharge tube. Note the one-to-one frequency correspondence between the signals of both, indicating the excitation of heavy particles within the plasma and ensuing propagation of an acoustic wave along the axis of the tube.

Correlation of signal amplitudes was not possible in the data of Fig. 4-8, since the transducer was not calibrated. Based upon previous designs, the microphone's first resonance lies somewhere between 900 c/s

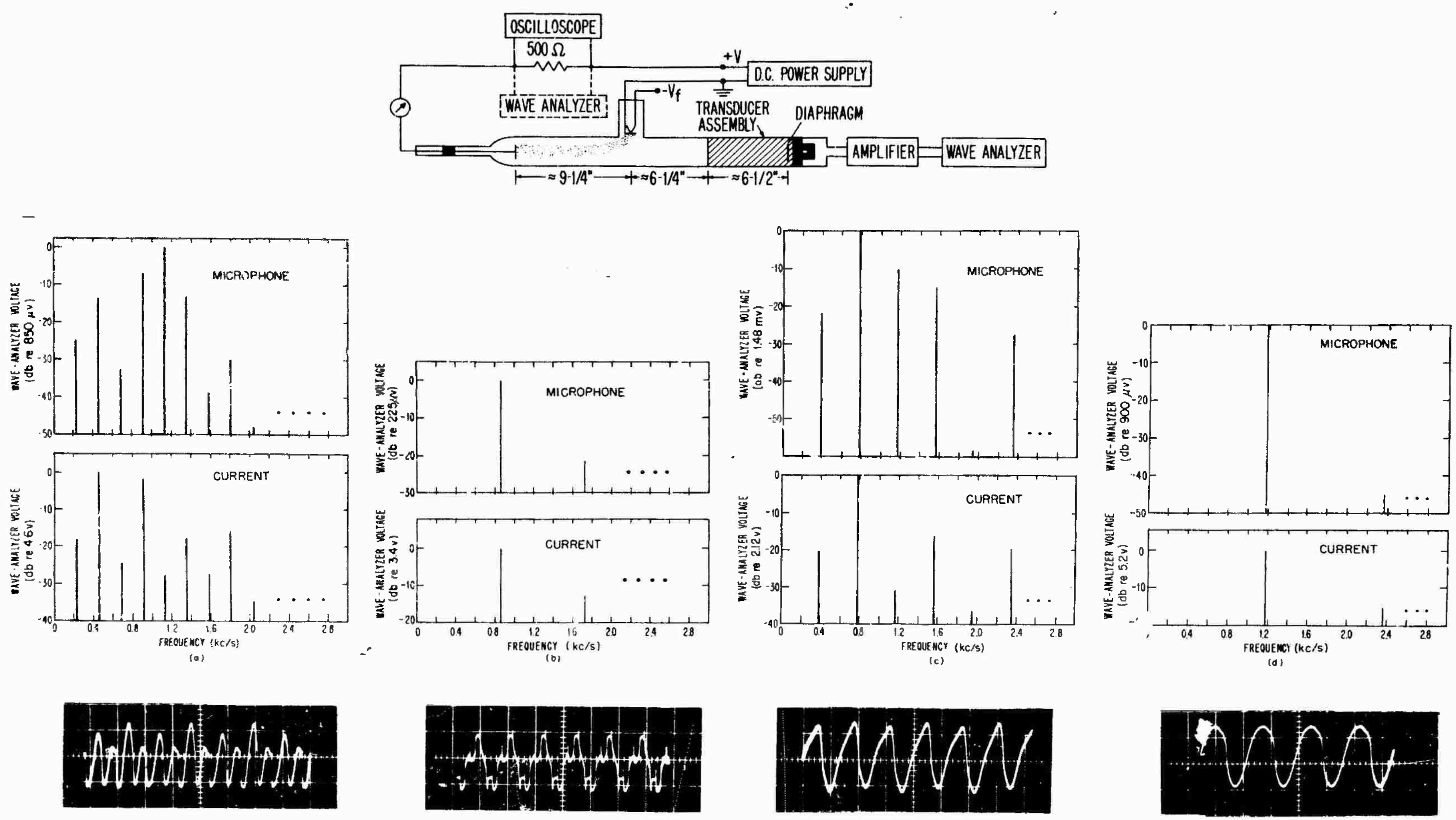

Fig. 4-8. Measured harmonic content of microphone responses and corresponding low-frequency oscillations in current for typical neon discharges. Data was taken using experimental configuration at top (a) Pressure = 1 Torr, average discharge current = 85 ma. (b) Pressure = 1 Torr, average discharge current = 66 ma. (c) Pressure = 5 Torr, average discharge current = 62 ma. (d) Pressure = 0.5 Torr, average discharge current = 76 ma. Horizontal scales in oscillographs of oscillation waveforms are 2 msec/major division in (a), 1 msec/major division in (b) and (c), 0.5 msec/major division in (d); vertical scales are 5 volts/major division in (a), (b), and (d), and 2 volts/major division in (c).

and 1200 c/s, predicting strong responses to the plasma oscillations within this region, a fact borne out in the measurements. Transducer sensitivity decreases faster above the resonance than below, and somewhere near 2400 c/s or 2500 c/s the unit becomes ineffective either as a microphone or loudspeaker in the simple detection scheme used. Amplitude correlations are further complicated by possible acoustic resonances in the space between anode and transducer.

Linear experiments on the artificial excitation of traveling variations in density support one contention that the coupling of energy from the ion wave to the neutral gas is insufficient to excite an appreciable sound wave in the neutral gas [1, pp. 56-57]. However, acoustic signals associated with the oscillations in the present research are sizable. Even with the crude microphone and straightforward detection scheme, wave-analyzer responses generally range between tens and hundreds of microvolts. In the region of the transducer resonance,it is even possible to observe plainly the sound waves on an oscilloscope after the 40-dB amplification.

4.6 Mixing Between Acoustic Waves and Spontaneous Oscillations

Previous attempts to excite acoustic waves in the discharge-tube plasma strongly suggested the coupling of such waves to the spontaneous low-frequency oscillations. In these cases, operation was always in a stable or quasi-stable regime of operation,which either made the effect of the plasma oscillation negligible or at most produced small-signal oscillations. In general, the build up and onset of such oscillations always produced an attenuation of acoustic waves transmitted into the discharge. When, for example, an acoustic modulation due to the loudspeaker was observed at the cavity output, its level gradually decreased as the strength of a natural oscillation grew in amplitude. Ultimately, the discharge would burst into a very large, but stable, oscillation and the signal corresponding to the modulation would settle around some constant minimal value.

To study this effect, various currents and pressures were established in a discharge such that large stable oscillations with diverse waveforms were formed. The transducer was operated as a loudspeaker at constant frequency and constant output intensity while the voltage across (current through) the 500-ohm resistor in the external circuit of the discharge tube was fed directly to the wave analyzer for a manual spectrum analysis. At the same time, the waveform of the low-frequency oscillation was observed across the same 500-ohm resistor with an oscilloscope. In all cases, regardless of pressure, current, acoustic frequency, or acoustic intensity, a frequency mixing was observed to take place between harmonics of the low-frequency oscillations and the acoustic waves emanating from the transducer. Figure 4-9 typifies such an interaction. It shows several cases where the small-signal sidebands of each harmonic in the oscillation occur at frequencies displaced from the harmonic by the exact amount of the loudspeaker frequency. Although the data shown were taken for only two convenient acoustic frequencies

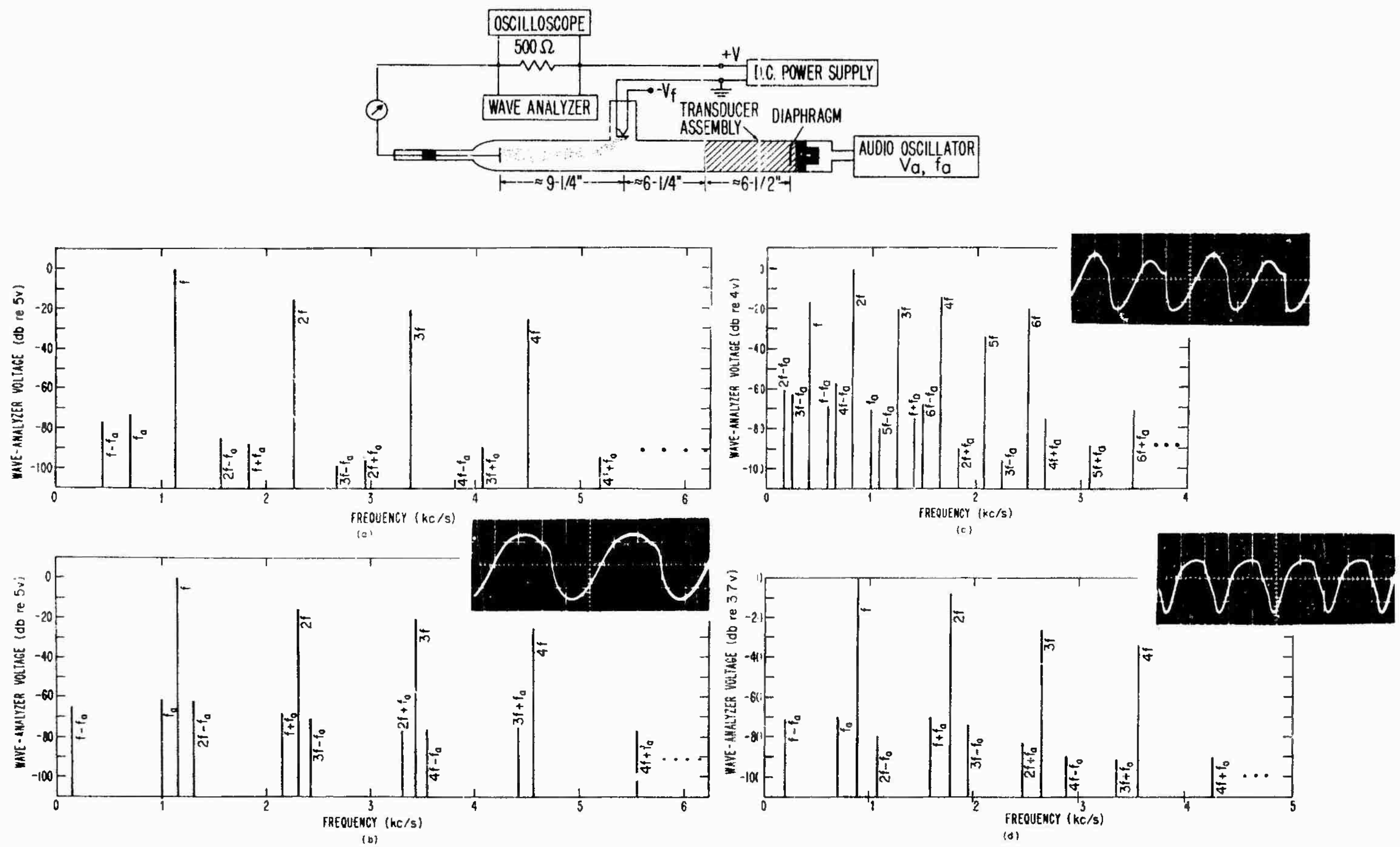

Fig. 4-9. Frequency spectra of current fluctuations in neon discharges which have been perturbed by sound waves from an acoustic transducer. (a) Pressure = 0.5 Torr, average discharge current = 70 ma, f_a = 700 c/s, V_a = 40 v p-p. (b) Same as (a), except f_a = 1000 c/s, V_a = 20 v p-p. (c) Pressure = 1 Torr, average discharge current = 80 ma, f_a = 1000 c/s, V_a = 20 v p-p. (d) Pressure = 0.4 Torr, average discharge current = 35 ma, f_a = 700 c/s, V_a = 40 v p-p. Oscillographs of current variations in the unperturbed discharges have horizontal scales of 0.2 msec/major division in (a) and (b), and 0.5 msec/major division in (c) and (d); vertical scales are 5 volts/major division in each.

– 700 c/s and 1000 c/s – similar results were found for any combination of loudspeaker frequency and plasma oscillation. Note that the frequency mixing is exact and that the sideband frequencies "fold back" when the oscillation frequencies are lower than the frequencies of the acoustic waves emitted by the transducer. In such a case, the wave analyzer responds to the absolute difference between the two frequencies.

Once again the fixed geometry in the setup prevents a correlation of signal amplitudes. This makes it difficult to speculate on the exact nature of the mixing process. It was clear that the amplitude of the sideband signals was dependent upon, and was controlled by, the loudspeaker output, but neither this parameter nor any other affected the mixing process as far as frequency was concerned. Furthermore, the intensity of the loudspeaker signal had no appreciable effect on the amplitude of the spontaneous oscillations. Although one might expect that coupling from the sound wave in the neutral gas to the ions might be large in a slightly ionized gas, this did not seem to be the case here since the low-frequency oscillations were excited quite independently of the loudspeaker output power and frequency, and remained unaffected by the activated transducer throughout the course of the experiments.

As a check on the frequency-mixing process, it was studied independently by RF probing of the discharge's positive column. Figure 4-10 again brings the interaction clearly in evidence for various discharge currents and pressures. In each of the cases presented, the RF frequency corresponds to resonance in the TE_{109} mode of the cavity. The correlation of amplitudes

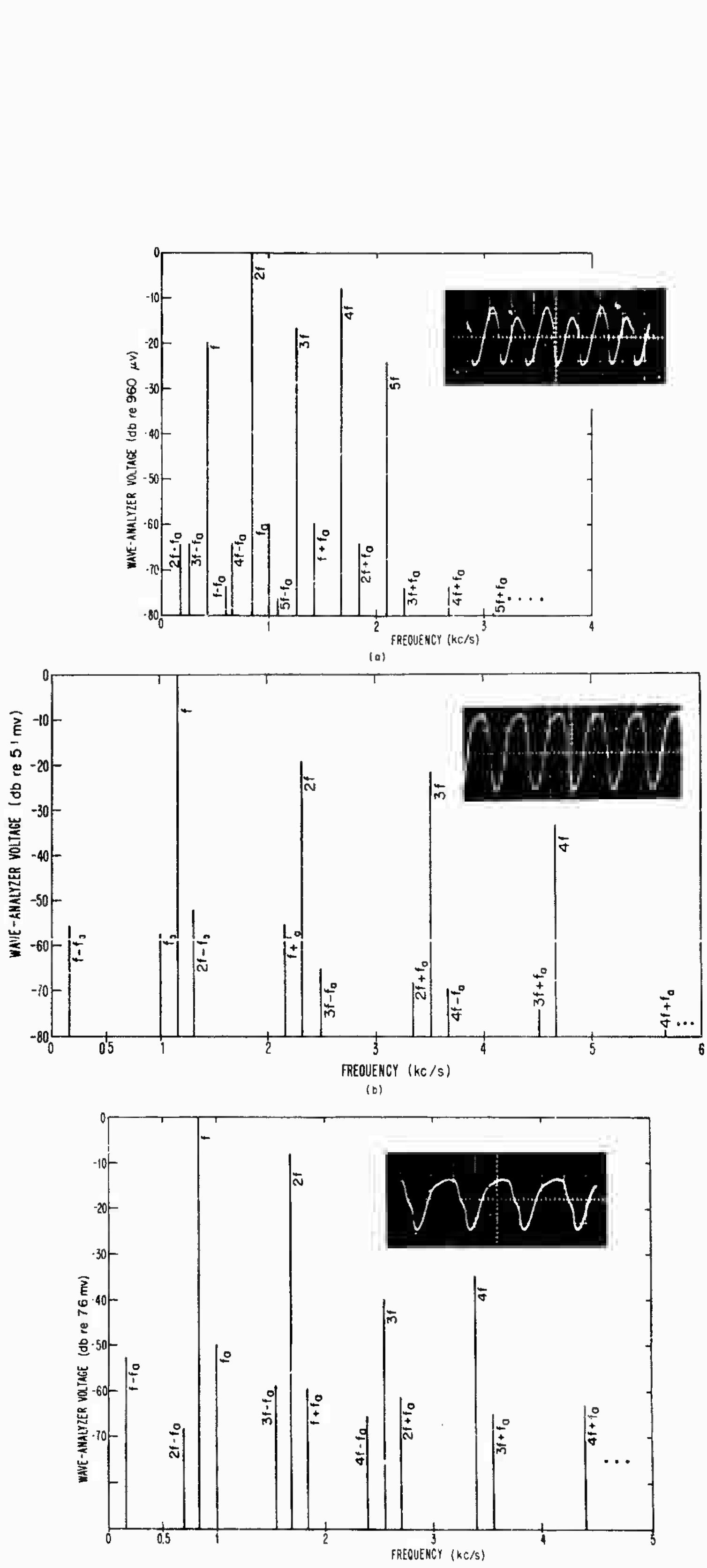

Fig. 4-10. Frequency spectra of fluctuations in the positive column of neon discharges measured with an RF cavity using the detection arrangement of Fig. 3-1. Cavity operation is near the resonance of the TE_{109} mode ($\approx$ 890 Mc/s). Oscillographs of current waveforms associated with fluctuations were taken across a 500Ω external load resistor. f_a = 1000 c/s, V_a = 25 v p-p. (a) Pressure = 1 Torr, average discharge current = 85 ma. (b) Pressure = 0.5 Torr, average discharge current = 70 ma. (c) Pressure = 0.4 Torr, average discharge current = 35 ma. Oscillograph scales: horizontal, 1 msec/major division in (a) and 0.5 msec/major division in (b) and (c); vertical, 5 volts/major division in each.

is almost impossible since the magnitude of the signals depends very critically on the RF. As with the excitation of sound waves in a quiet plasma, the wave-analyzer voltage for a fixed sound intensity depends upon the slope of the Q-curve. Since the large spontaneous oscillations may even be removed from the small-signal category (which served as the basis for connecting wave-analyzer response to the slope of the Q curve), they presumably result in nonlinear and distorted cavity outputs which, when fed to the wave analyzer, would give misleading values. Regardless of the possibility that nonlinearities occur, no sidebands other than the first-order ones of each component in the oscillation were observed in either the RF probing or in the previous similar measurement of fluctuations in the discharge-tube current itself.

4.7 Additional Observations

During the course of the main work, the moving striations sometimes manifested themselves in interesting ways. For example, there were several combinations of current and pressure for which spontaneous oscillations could either be created or eliminated by proper positioning of the movable anode. In fact, this provided one convenient way of manipulating the striations in and out of particular experiments; but, beyond that, this observation posed some questions about the very nature of the oscillations. A superficial examination of the anode position relative to the onset or disappearance of the moving oscillations revealed that the fluctuations could be made to disappear, then reappear, then disappear again over total anode traversals of 2-3 inches. Although the wavelength corresponding to the observed oscillation frequencies in the neutral neon gas differed greatly from this dimension, velocities of the moving striations themselves have been reported between 10 and 1000 m/s. For the lower values, wavelengths corresponding to the frequencies which were observed in the discharge tube could be in the 2-3" range. This suggests the possibility of some connection between the traveling ionization waves and the "tuning" of the positive column with the anode. However, the inflexibility of the original configuration for an experiment along these lines (primarily because of limited anode motion), did not permit the accumulation of enough data to allow any logical conclusions about this effect.

Attempts were also made to excite uncontrolled spontaneous oscillations in the discharge tube by the brute-force propagation of high-intensity sound waves into the discharge. By appropriate adjustment, the discharge was made to operate in suspected quasi-stable regions of the current-pressure plane before activating the transducer, whereupon the loudspeaker was activated at various intensities and frequencies. In many cases, moving striations were excited, or at least seemed to be excited, upon activation of the loudspeaker. On certain occasions, however, oscillations did not appear, possibly because the discharge tube was too far into its stable region. Again, the experiment was inconclusive because – at best – the region of operation was based on guesswork. Techniques for exciting and observing traveling ionization waves (such as that used by Cooper in his work) would be necessary here before one could make any concrete observations concerning the influence of an externally induced sound wave in exciting large-signal oscillations.

By changing from a continuous dc discharge to a pulsed dc discharge, and by introducing a simple IF detection scheme, it was also possible to take a brief look at the effect, if any, of sound waves emanating from the transducer on the Tonks-Dattner plasma resonances frequently observed in afterglow plasmas [14, 15, 16]. A result of transverse electroacoustic eigenmodes in the plasma, these resonances are supposedly sensitive to electron density, electron temperature, and indirectly to neutral gas temperature. Therefore, it seemed logical to expect that variations in these parameters caused by the propagating sound waves might have some effect on the resonances.

To look into this, a section of a narrow strip transmission line about 1/2" wide was bent to conform to the shape of the cylindrical discharge tube and placed around it in the positive column so that the tube was transverse to the axis of the transmission line. The strip was driven by a heavily padded RF oscillator through a directional coupler, and the reflected signal was mixed with a properly tuned local oscillator in a mixer rectifier whose output was amplified and observed on an externally swept oscilloscope. Discharges at pressures from 0.1 to 2 Torr were pulsed at rates of about 100 pps resulting in the observation of 3 or 4 resonances – all within a few milliseconds in the afterglow. When the loudspeaker was activated, there was no observable change in either the amplitude or the position and spacing of the resonances. This was the case regardless of loudspeaker output power and frequency.

Chances are, if there were any changes in the resonances, they would not have been detected with the relatively insensitive scheme that was employed. Bearing in mind that previous experiments and estimates placed the possible relative pressure variations caused by the loudspeaker at about 10^{-4}, and assuming that the relative temperature changes in the neutral gas and the relative variations in electron density were about the same, it does seem unlikely, in retrospect, that there would have been a pronounced indication of such an effect.

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5. SUMMARY

Measurements in this research demonstrate the feasibility of acoustically modulating both the electron-plasma frequency and electron-collision frequency of a weakly ionized gas, and observing the resultant effect with a simple RF detection system. Although coupling to electrons is relatively inefficient using sound transducers at reduced pressures, the data indicate that low-frequency pressure fluctuations of approximately 10^{-5} Torr in the 0.1 - 0.3 Torr ambient pressure range cause plasma-frequency perturbations in the order of 10^{-4} times the plasma frequency for plasma frequencies up to 1000 Mc/s. Corresponding changes in collision frequency have not yet been determined, but should follow in a straightforward manner by means of the procedures outlined for the determination of Δf_{pc}.

Once an appropriate transducer was designed and developed for incorporation within the discharge tube, natural sound waves generated by the discharge-tube plasma itself were also easily observed. Data taken with the transducer operating passively as a microphone established a definite link between sound waves detected in the neutral gas and the spontaneous low-frequency oscillations generated by the discharge (those commonly referred to as moving striations). An effect which is apparently linked to these oscillations manifested itself in their apparent coupling to the sound wave generated in the positive column of the discharge by the transducer when the latter was operated as a transmitter. As a result, moving striations were purposely excited in the plasma to determine the possibility of an interaction between variations in particle density created by them and the perturbations produced by the external sound source. A frequency mixing process between the two was clearly confirmed when the acoustic waves emanating from the transducer were seen to create first-order sidebands of each harmonic component of the complex periodic oscillations.

Both the solid-dielectric and the bipolar moving-armature types of transducer have proved adequate as sound sources for studying the plasma modulation effect. The moving-armature type has a greater output power in the audio range. Each design has successfully withstood the high temperatures and low pressures consistent with good vacuum-processing techniques. In addition, both units have a potential application for use as microphones in investigating sound waves or other low-frequency oscillations generated by the plasma itself. Unfortunately, operation of either transducer at the reduced pressures encountered in laboratory plasmas degrades their acoustic characteristics in a manner which is difficult to predict quantitatively. Therefore, a careful study of transducer properties in a low-pressure environment would certainly be in order before accurate estimates of performance in the plasma could be made. However, because of their compactness and placement within the discharge tube, both types are ideal as probes since neither disturbs the electrical properties of the discharge itself.

On the basis of the correlation between theoretical and experimental values of $\Delta f_{pc}/f_{pc}$ in the linear portion of the curves in Figs. 4-5 and 4-6, it seems reasonable to conclude that the relative change in electron density

$(\Delta N_e/N_e)$ does indeed equal the relative change in neutral molecule density $(\Delta N/N)$ for slightly ionized gases which are subjected to low-frequency pressure variations. Experimental evidence is now available which is well explained in terms of this relationship in two cases: (1) constant plasma frequency with variable acoustic intensity (Fig. 4-3), and (2) variable plasma frequency with constant sound intensity (Figs. 4-5 and 4-6). The observations also support the "frozen composition" assumption of Sodha and Palumbo that, if the relaxation times of the ionization and de-ionization processes in the plasma are much greater than the period of an acoustic cycle, then the composition of the gas remains practically unchanged during such a cycle [1, p. 1636].*

Two immediate steps were taken in an effort to explain the dips that occurred in the data of Fig. 4-5. One was to expand the acoustic range toward higher frequencies (Fig. 4-6); the other was to make measurements in a region around a higher cavity resonance (Fig. 4-7). Investigations based on both approaches indicated that the effect was a product of the medium itself and was quite independent of the acoustic and RF parameters. Because of the subsequent observation of nonlinear mixing between frequencies emanating from the loudspeaker and those present in discharges with uncontrolled low-frequency oscillations, it was concluded that the sound perturbations created

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*An alternative limiting assumption about the composition of the gas is that the temperature of all the species in the gas at a given point and time is the same. In this case the ionization and de-ionization rates are so fast that at all instants the gas is considered to be in thermodynamic equilibrium, i.e., the time of relaxation of the ionization and de-ionization processes is much less than the period of an acoustic cycle. Assuming that the change of state of the neutral gas is adiabatic, it can be shown that $\Delta N_e/N_e = (\text{constant})(\Delta N/N)$ when the plasma has this "equilibrium composition" (1, p. 1637). In practice, this condition could be realized in pulsed discharge-tube plasmas after sufficient times in their afterglow.

by the loudspeaker couple to such plasma oscillations even when the latter appear to be absent from the discharge. Such could be the case if the discharge were in a quasi-stable condition at plasma frequencies favoring the build up, and perhaps the onset, of oscillations, especially under the influence of a substantial mechanical disturbance in the plasma.

At this point, experimental investigations should proceed in two separate directions. First, a well-behaved plasma should be constructed which has no instabilities or striations for further quantitative measurements of plasma-frequency and collision-frequency variations under the influence of a sound perturbation. Until recently, it was all but impossible to build a quiet laboratory plasma with these properties. However, Persson at the National Bureau of Standards has developed a novel cold-cathode discharge [2] which takes advantage of a complex "brush" cathode that generates a large uniform electron beam with a corresponding negative glow which has a longitudinal dimension one or two orders of magnitude larger than for the normal cathode. The negative glow is essentially field-free and recombination-dominated, making it a practically uniform plasma. Its large dimensions, minimal noise levels, and the fact that large electrical oscillations do not occur, make it ideal for investigations involving the excitation and detection of acoustic waves in plasmas. Physically, the "brush" cathode discharge can be constructed in exactly the same T-shape configuration used in the experiments reported in the preceding pages, and even lends itself to the same method for mounting acoustic transducers. Wide control over the medium offers the opportunity to make conclusive measurements on sound perturbations within broad ranges of pressures and discharge-tube currents. The only additional precaution that one must take is to use minimal RF probing power since the brush-cathode plasma is a low-temperature medium which can easily change its characteristics when subjected to RF heating.

As a second area of continued research, conventional laboratory discharges—which, more often than not, generate uncontrolled low-frequency oscillations—should be probed with a calibrated transducer to establish more information on the moving striations. Even with the endless work on these fluctuation phenomena, a full understanding of their mechanism of generation is not yet available. In the last few years, however, great advances have been made in the development of a theory to explain the oscillations, and the results that have thus far been produced indicate a perturbation effect in the neutral gas. Therefore, a sound transducer which excites or detects neutral particle disturbances would be an ideal diagnostic tool for either confirming or suggesting theories for the traveling ionization waves. As for the two transducers already built, the solid-dielectric type is immediately applicable as a microphone for determining the relative amplitudes of sound signals generated by the oscillations since it has a very flat frequency-response characteristic at frequencies up to its resonance, which, at pressures of about 1 Torr, is estimated to be around 10 kc/s – well below the acoustic frequencies observed thus far in the neon discharges. One immediate task, then, is to substitute this transducer for the moving-armature type used in the initial measurements in order to extend the observed frequency correlation between sound waves and moving striations to one that includes the relative amplitudes of the two.

Of course, there is always the possibility that other types of plasma oscillations can be detected, perhaps at higher frequencies. Presumably, these would create much smaller sound intensities than the moving striations and would require a much more sensitive detection system. However, the solid-dielectric transducer may still be adequate in its present configuration for this purpose.

It would be interesting to replace the movable anode in the discharge tube with an acoustic transducer. In this case, the diaphragm of the transducer could double as the anode, assuming that the remainder of the structure was properly biased electrically to prevent irregularities in the discharge. Using this technique, one would then have a transmitter and receiver in the same discharge. This would eliminate some of the guesswork in establishing the acoustic intensities at various points along the discharge tube. Instead of referring the propagation of acoustic waves to a transducer assembly that is far removed from the point at which the positive column is probed, sound pressure, particle displacement, and the other parameters associated with sound wave propagation could be determined at the anode end. Note that the microphone would also serve as a perfect reflector of sound waves impinging on it. In addition to providing more accurate descriptions of standing wave patterns in the discharge tube, this arrangement might also be potentially useful in making dispersion measurements on sound waves in plasmas and in determining the influence, if any, of particle density variations in the ionized medium on the acoustic wave propagation.

With the relatively simple and controllable laboratory apparatus described here, one can study the interaction between sound and longitudinal plasma waves, possible modification of the nonlinear mixing process between two RF frequencies propagating through a plasma, and the potential of combining sound with electromagnetic waves for plasma diagnostics. An interesting application that comes to mind is the determination of sound speed – hence, neutral-particle temperature – in a plasma. This possibility could be demonstrated in the present configuration by pulsing the loudspeaker, and precisely measuring incident and reflected pulses through observation of the RF modulation. However, the implementation of this experiment would require a wideband transducer and a detection system with a larger bandwidth than that now available, posing an additional problem of increased plasma-noise levels.

The possibility of propagating electromagnetic waves through a cold isotropic plasma at frequencies below the plasma frequency, as suggested by Drummond [3], is intriguing. For an experiment aimed at verifying Drummond's theory, initial calculations indicate that the cavity used in the preceding measurements would not be suitable as the source of localized transverse electromagnetic waves required by the geometry of the proposed model. For a neon discharge with a plasma frequency typical of those reported in this research, an acoustic disturbance of the ions at a frequency of about 300 kc/s would require an RF in the vicinity of 100 Mc/s before the Drummond effect could be observed – if, in fact, it does exist. This RF is well below the cutoff frequency of the present cavity so that an alternative technique has to be found. One possible approach is to place a balanced two-wire transmission line with closely spaced, filamentary conductors through the positive column of the discharge, and transverse to the axis of the tube. Such an arrangement offers a broadband probe which allows flexibility in seeking and using an appropriate RF. On the other hand, it has the potential drawback of disturbing the plasma itself, as would any probe which was placed in the path of a discharge. However, such a configuration has been constructed and measurements are now in progress to determine its effect on the positive column. If its influence on the plasma is minimal, or can be made negligible by proper biasing, either a solid-dielectric transducer or a piezoelectric quartz transducer will be used in conjunction with the line to investigate Drummond's prediction.

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13. ABSTRACT

This report contains initial results of an experimental study of acoustic disturbances in weakly ionized gaseous plasmas which exist in the positive column of dc discharges. One phase of the research concerns itself with the effects of externally perturbing such a medium with acoustic waves emanating from transducers that operate in the audio and ultrasonic frequency ranges. The balance of the effort is devoted to the observation of sound waves generated by natural and uncontrolled low-frequency oscillations which commonly occur in laboratory discharges.

Compact transducers incorporated into cylindrical discharge tubes provide the source of acoustic signals and serve as microphones in detecting sound waves. Modulation of electromagnetic waves by the intentionally disturbed plasmas was measured in a novel rectangular cavity whose output was detected and fed to a wave analyzer. Resultant wave-analyzer responses indicate that the electron collision frequency was modulated in addition to the electron plasma frequency, and that the variations in both were proportional to the magnitude of loudspeaker diaphragm deflection, as predicted by simple acoustic theory. Measurements indicate that $\Delta N_e/N_e = \Delta N/N$ (where N_e and N are electron and neutral molecule densities, respectively) for slightly ionized gases which were subjected to low-frequency pressure variations in the order of 10^{-5} Torr at ambient equilibrium pressures in the 100-300μ range, and that such variations produced plasma-frequency perturbations of 0-140kc/s at plasma frequencies up to 1000 Mc/s.

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ABSTRACT (Continued)

Early observations of intentional acoustic perturbations were made in stable and quiescent discharges over a pressure range for which large spontaneous oscillations in the positive column were absent. However, frequent and unpredictable appearance and disappearances of moving striations made it extremely difficult to maintain a quiet plasma and, as measurements progressed, there were indications that the plasma was often in a quasi-stable condition at which time the sound waves transmitted into the medium by the transducer actually excited a low-frequency oscillation and interacted with it. Ultimately, a separate investigation was conducted to determine the nature of the interaction between the sponanteous plasma oscillations and the sound waves generated by the transducer. In addition to an attenuation of the sound waves propagating into the plasma, a clear cut frequency mixing was observed between the low-frequency oscillations and the forced sound wave.

Finally, the transducer was used as a microphone without the RF circuitry to "listen" to the sound waves created by the moving striations in the neutral gas. Although it was impossible to correlate the magnitude of the low-frequency oscillations with the intensity of the sound waves produced by them, a one-to-one frequency correspondence was in evidence at all times. Thus, a link was established between the electrical oscillations in the plasma and acoustic waves in the neutral gas; and, a technique was demonstrated for quantitatively investigating the relationship between the two.